Social Security, Endogenous Fertility and the Optimal Family Size

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Abstract: This note analyzes a model of endogenous fertility in the presence of financial market assets and social security pensions. Given the children externality, the fertility rate chosen in a market economy is too low compared to the Social Optimum, asking for a corrective policy. Indeed, the representative household does not take into account this children externality which leads to a suboptimal family size. We show that an optimal demographic allocation can be implemented through a subvention taxation policy.

JEL classification: J13; H55; H25
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1 Introduction

In the literature on population growth, two main dominant views of reproductive motivations are identified (Dasgupta, 1993). In the first one, children are considered as durable consumer goods. In the second one, and particularly in developing countries, children are mainly viewed as investment goods, hence parents reproductive motivation is thus associated to the so called old-age security hypothesis (Leibenstein, 1957). According to the latter (Jellal, 2000; Jellal and Wolff, 2002a), in a socio-economic environment where parents are unsure about their potential ability to support themselves during the old age, they may rear children in the expectation of receiving support from their children in their old age (Neher, 1971; Willis, 1980; Nerlove et al. 1987). The underlying transfer mechanism is one of direct reciprocity driven by evolutionary cultural transmission, where parents give first and children give later, without any intergenerational conflict. Children are implicitly considered as passive agents under the control of parents, who honor the contract loan by reimbursing their parents during their old-age. This transfer mechanism is self-enforcing.

Indeed, in the literature explaining inter vivos transfers within the family, Jellal and Wolff (2002b) have proposed an evolutionary cultural model of filial altruism formation. Moving away from the altruism and exchange hypotheses, the authors study the endogenous altruism approach in which private transfers are rooted in a purposeful shaping of preferences within the family. The purpose of their paper is to analyze the motives underlying upstream inter vivos transfers between the generations by departing from the traditional approach suggested by Becker (1993), based on the hypothesis of exogenous altruism and exchange reciprocity. Jellal and Wolff (2002b) have instead presented a model of endogenous altruism, where private transfers are aimed at shaping preferences within the family. Rather than relying on the so-called demonstration effect theory, they have presented a model of cultural transmission of filial altruism in which individuals can be of two types, altruistic or non altruistic with respect to their parents, and the probability of a child being altruistic depends positively on the amount of upstream care shown by parents. Mainly, they show that only the altruistic type of preferences survives evolutionary selection, and hence the cultural system converges in the long run to a population having kin obligations. Moreover, the old-age security hypothesis has been strongly supported by many empirical studies in developing countries, either in Africa or in Asia (Nugent, 1985). For instance, in the case of Malaysia, Raut (1996) shows that the probability that parents will rely on the support of their children when they are old is significantly lower for households having high income, with

\[ \text{This term refers to the obligations in terms of family responsibilities in general and especially to older relatives (filial obligations). For the use of this term see for example Hoff and Sen (2005).} \]
better access to private pensions and other financial assets. Therefore, we infer that, when parents consider children as investment goods, the development of social security policy and financial markets is expected to lower fertility and leads to a demographic transition. Indeed, Nugent and Gillaspy (1983) and Entwisle and Winegarden (1984) document that the demand for children tends to decrease with public pension programs development.

Hence, it seems that the presence of social security tends to substitute public for private transfers as a support for old age and thus provides motivations to have fewer children. However, having few children per household creates a large negative externality on the social security pension. Indeed, when deciding to choose optimally the size of its family, the representative household does not take into account this externality; this behavior may be harmful for the viability of a social security system in a decentralized economy. Indeed, in almost all countries, the financing of public pension systems is a pay-as-you-go plan. In this case, their financial viability depends on the dependency ratio of the elderly. This ratio has experienced a sharp decline for decades and will continue to fall. According to the medium variant of the United Nations projections, the proportion of people aged over 60 will rise from 10.3% in 2005 to 21.8% in 2050 in the world. In developing countries, the demographic transition will be more severe and the ratio, for the same dates, rise from 8.1% to 20.1%. This aging population that threatens the financial viability of pension systems has two causes: longevity and fertility decline. According to the same projections of the United Nations medium variant, the total fertility rate was 4.75 children per woman in the Seventies to 2.02 in 2050. These figures are even more alarming in developing countries over the same period as the composite index decreased from 5.41 to 2.05.

The present note is a contribution to the endogenous fecundity theoretical literature. It presents a simple two-period model with endogenous fertility and social security. Its main objective is to show that, when parents consider children as investment goods, the family size prevailing in a market economy is too low compared to the Social Optimum. Intuitively, when the viability of the social security system depends on the average fertility rate, optimizing households do not take into account this externality and, consequently, the family size turns out to be suboptimal. Then, we show that an appropriate taxation policy may correct this distortion and lead to the optimal demographic allocation.

The rest of the paper is organized as follows: Section 2 presents a simple model. Section 3 analyzes the fertility choice under a market economy, whereas the implementation of the optimal centralized fertility allocation is presented in Section 4. Finally, Section 5 concludes.

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2 The Model

We consider a simple two-period model economy. Economic agents maximize an inter-temporal utility of their current and future consumptions. Young agents work, earn labor wage and by filial obligation and cultural transmission, provide an exogenous private transfer to their old parents. Following the old-age hypothesis, we suppose that young individuals rear children as investment goods. Although their fertility is costly in terms of forgone labor income, when they become old, they receive a transfer from their children. We suppose that the government subsidizes their rearing cost by balancing its budget through a flat tax. Moreover, to be eligible for social security pension, young agents pay a proportional tax on their labor income, which is used by the government to finance a public pension system. Finally, young invest savings in the financial market to smooth their consumption profile.

The representative young agent maximizes the following inter-temporal utility function:

\[ V = U(c_t) + \beta U(c_{t+1}) \]

where \( c_t \) and \( c_{t+1} \) represent respectively the consumption profile in youth and in old age for an individual born at time \( t \); \( \beta \) is the standard discount factor. This representative agent maximizes her utility, taking into consideration the following budget constraints:

\[
\begin{align*}
  c_t + s_t + T_t &= w_t(1 - \theta - m - (1 - \tau)\varphi(n_t)) \\
  c_{t+1} &= w_{t+1}n_tm + (1 + r_{t+1})s_t + p_{t+1}
\end{align*}
\]

where \( \theta \) is the pension contribution fixed rate paid by all young individuals, \( mw_t \) is the transfer to the parents, while \( \varphi(n_t) \) denotes the fertility convex function cost or the cost of raising \( n_t \) children, which is assumed to be subsidized at a fixed rate given by \( \tau \). \( s_t \) is the amount of savings and \( T_t \) is a flat tax collected by the government to finance the granted subvention. In the second period of her life, the representative agent receives a gross interest factor \( (1 + r_{t+1}) \) on her optimal savings. Further, according to the self-enforcing old age norm, the agent receives also a fixed transfer from each of her children \( w_{t+1}n_tm \) and a public pension of value \( p_{t+1} \).

The government budget constraints are shaped as follows:

\[
\begin{align*}
  T_t &= \tau w_t \varphi(\bar{n}_t) \\
  p_{t+1} &= \theta w_{t+1} \bar{n}_t
\end{align*}
\]

where \( \bar{n}_t \) is the ex-post optimal average fertility rate in the total population. This represents an intra-generation externality.
3 Fertility Under a Decentralized Market Economy

By taking factor prices, and the average number of children as given, the representative individual of generation $t$ chooses material consumption over the life cycle and fertility to maximize her lifetime utility function under budget constraints:

$$\begin{align*}
\text{Max} \quad V &= U(c_t) + \beta U(c_{t+1}) \\
ct + st + T_t &= wt(1 - \theta - m - (1 - \tau)\varphi(nt)) \\
c_{t+1} &= wt_{t+1}ntm(1 + rt_{t+1})st + pt_{t+1} \\
\text{Ex-post} \quad nt &= \bar{n}_t
\end{align*}$$

The first-order conditions for an interior solution for savings and fertility rate are given respectively by:

$$\begin{align*}
U'(c_t) &= \beta(1 + rt_{t+1})U'(c_{t+1}) \\
w_t(1 - \tau)\varphi'(nt)U'(c_t) &= \beta mw_{t+1}U'(c_{t+1})
\end{align*}$$

**Proposition 1** The optimal number of children of the representative household is given by $^2$

$$\varphi'(nt) = \frac{m}{w_t(1 + rt_{t+1})(1 - \tau)}. $$

**Proof** It is easily obtained by substituting equations (2) and (3).

Since the representative household considers children as investment goods, then according to Proposition 1, its optimal family size depends positively on the return to investing in children, and thus the private transfer dictated by filial obligation (Jellal and Wolff, 2002b), as well as the growth of wage, but negatively on the return to its optimal saving option. Further, since the cost of raising children is subsidized, this incentive to procreate encourages the representative household to have more children. To conclude, depending on the extent of the taxation subvention scheme, the optimal number of children may be too low since the household does not take into account the children externality given by the average fertility. The availability of an appropriate taxation policy may lead to an optimal demography which internalizes the impact of population on the sustainability of social security pension.

4 Fertility Under a Centralized Economy

In order to the sub optimality of demography under a decentralized market economy, we consider the fertility problem which is implemented by a

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2 The proof is easily derived from the FOCs of the maximization program.
benevolent planner. The main role of the latter is to recognize the population externality impact on the social security system’s viability. This recognition leads the planner to internalize the positive children externality and choose the optimal fertility rate. Indeed, a benevolent planner is supposed to maximize the representative agent’s utility, and thus solves the following program:

\[
\begin{align*}
\text{Max} & \quad V = U(c_t) + \beta U(c_{t+1}) \\
\quad c_t & = w_t(1 - \theta - m - (1 - \tau)\varphi(n)) - s_t - T_t \\
\quad c_{t+1} & = w_{t+1}n_t(m + (1 + r_{t+1})s_t + p_{t+1}) \\
\quad T_t & = \tau w_t\varphi(\bar{n}_t) \\
\quad p_{t+1} & = \theta w_{t+1}\bar{n}_t
\end{align*}
\]

Ex-post \, \, n_t = \bar{n}_t

The first-order optimal conditions for an interior solution for savings and fertility rate are given respectively by:

\[
\begin{align*}
U'(c_t) & = \beta(1 + r_{t+1})U'(c_{t+1}) \\
\varphi'(n_t)U''(c_t) & = \beta w_{t+1}(m + \theta)U''(c_{t+1})
\end{align*}
\]

From these optimal equilibrium conditions we obtain the optimal fertility rate.

**Proposition 2** The optimal number of children chosen by a social planner is given by:

\[
\varphi'n^*_t = \frac{w_{t+1} + m + \theta}{w_t(1 + r_{t+1})}
\]

We observe immediately that the fertility rate arising in a centralized economy differs from that chosen in a decentralized economy. Therefore, now, the optimal number of children depends on the fixed rate of pension contribution given: the higher this contribution, the higher number of children.

Given this first best allocation, we now examine how the government can use the available taxation policy to achieve the social optimum.

**Proposition 3** There exists a tax-subvention scheme leading to the optimal fertility rate. This tax amounts to:

\[
\tau^* = \frac{\theta}{m + \theta}
\]

We observe that the optimal tax subvention rate only depends on the transfers’ parameters, either those within the family or those provided by social security pension. Moreover, the optimal tax is raised by the social security contribution rate and lowered by the size of private transfers within the family.
5 Conclusion

In this note we have analyzed a simple model of endogenous fertility in the presence of financial market assets and social security pensions. Given the presence of a children externality, the fertility rate chosen in a market economy is too low in the absence of corrective policy. Indeed, the representative household does not take into account this children externality which leads to a suboptimal family size. We have finally shown that an optimal demographic allocation exists and can be implemented through a subvention taxation policy.
References


