Politicl Regime Change and State Performance

Debojyoti Mazumder* Rajit Biswas
Indian Institute of Management Indore Centre For Development Studies

Abstract: The present model analyses how the state would provide services when the change of power depends upon the performance of the state. Agents can evaluate state performance based either only on the receipt of government services, or both on the benefit from government services and taxes imposed. With a credible threat of power change, if the valuation of the government services is low, along with a low fiscal capacity, then it is less probable that this service would be provided. Furthermore, such an allocation is compared with a situation, when there exists a threat of active opposition. Interestingly, that threat does not change the optimum provisioning of government services (as compared to the previous situation) in the equilibrium.

JEL classification: D72, D74, H2, H4
Keywords: State capacity, regime change, conflict, taxes, government services

The authors are thankful to Dr. Neelanjan Sen, Assistant Professor, St Xavier's College, Kolkata and Dr. Satyaki Mazumder, Assistant Professor, IISER, Kolkata for their helpful comments. They are also grateful to two anonymous referee for their suggestions, that has helped to improve this paper. The usual disclaimer applies. This study is not funded by any agency. The authors declare that they have no conflict of interest.

* Address: Prabandh Shikhar, Rau-Pithampur Road, Indore, 453556, Madhya Pradesh, India, Email: debojyotim@iimidr.ac.in).

Recommended Citation

Copyright © 2017 University of Perugia Electronic Press. All rights reserved
1 Introduction

Modern states are often designated as “welfare states”. Often, the duty of the welfare state is to provide government services, which typically markets cannot provide efficiently. For example, neo-classical economists argue that markets fail to provide public goods efficiently, and hence state should provide such goods. Meanwhile, most of these modern welfare states are “democratic” and are characterized by various political parties, competing among themselves to capture power. Government service provisioning can affect election outcomes (for example, see Pierre and Sandrine 2011), and it is often the case that, in a democracy, one of the criterion by which the electorate judges the performance of these parties (and thereby their worth) is their relative effectiveness in providing such service. States that may fail to provide various welfare services can land itself in a situation of social unrest. On a similar note, there is a prominent literature in political science where the role of public provisioning in electoral decision (Greasley et al. 2011, Gherghina 2011, etc.) is discussed. Indeed, in a society with ethnic, class or any other division that becomes the basis of deprivation, conflict can be an obvious outcome. The present contribution tries to capture this facet of a modern welfare state by developing a very simple model.

In a world inhabited by two distinct groups, when each individual of either group values the service provided by the state, political competition among these two groups can yield some very striking conclusions. As argued above, political regime change, in this model, depends crucially on the government’s performance in providing the service. The incumbent leader does not have any information about the actual identity of the people (and hence the measure of its support), who would support the ruling party during change of power. However, the incumbent leader believes that the probability of winning depends upon its fiscal policy (i.e. on its decision of service provisioning). On this note, we develop two different models, one with “partially rational agents” and the other with “fully rational agents”. In the former set up, agents only care about the amount of service received, and does not evaluate the role of government as an efficient provider of

---

1 In the words of Milton Friedman: “It can be argued that private charity is insufficient because the benefits from it accrue to people other than those who make the gifts ... I am distressed by the sight of poverty; I am benefited by its alleviation; but I am benefited equally whether I or someone else pays for its alleviation” (see Friedman 1962).

2 See Howard (2014). Gurr (1968) argues in his book “Why Men Rebel” that relative deprivation is the most often cause of rebellion. We do not intend to posit that relative deprivation among two or more groups can only be in terms of welfare services received from the government. However, since the state can directly affect the economy in a capitalist society by allocation of the public goods, this indeed is a very plausible channel that can cause social conflicts.

3 Agents strictly speaking are the voters, or more precisely the constituents. Our choice of the nomenclature is keeping with the usage in the literature of political economy.
public good. In the second model with fully rational agents, the incumbent leader believes that the agents are more conscious, and would only support the government, if they are better off vis-a-vis a situation where there is no government. Here, government has no role but to redistribute. The agents would support only if they get more than their private income by supporting the incumbent leader. The belief structure of the incumbent leader is thus different in the two cases, and neither of the situations are fully deterministic. While the exercises are pretty different, the equilibrium results are quite close. If people have a very high valuation for the government service, then the service will be provided in its full capacity (which will be financed by taxes that would be charged at the maximum level). If the agents’ demand for the service is low, no service will be provided at all. More interestingly, maximization of expected utility by the incumbent leader entails, that in such a situation the benefit of the subsidy will be provided to the opposition. This is because at the equilibrium a very low level of government service has been provided, it is highly probable that the incumbent leader would lose the elections. Thus, in the next period taxes levied on the opposition may become applicable on the incumbent leader with a sufficiently high probability. Although the second result is also generated when agents are fully rational, but in that case no equilibrium value can be determined (in terms of policy variables). However, optimal policy direction can always be specified. This is discussed in some details in Section 2.3.

In another variant of the model, the role of a more pro-active opposition is discussed. Note that the previous version of the model, does not include any active participation of the opposition leader. The probability of losing power, as faced by the ruling incumbent leader, depends only on the performance of the incumbent leader, as evaluated by its ability to deliver government service (its failure to do so satisfactorily can be viewed as a sort of anti-incumbency). It is now assumed that the opposition may develop costly action termed as “arms” and initiate a “violent revolution” (the term is used only for ease of nomenclature). This can reduce the probability of winning, for the incumbent leader. In case the agents do not value the service much, the opposition is going to get the subsidy. Moreover, to have a situation of armed conflict, both the valuation of government service by the agents and the marginal impact of arms on the probability of power change should be high simultaneously. Otherwise, the opposition leader does not have sufficient incentive to capture power in the next period through investing in arms. Thus, the opposition necessarily does not try to enforce the state to provide the public service, and will choose to revolt only in those situations where the marginal benefit from government service is very high. This can be corroborated with the fact that armed conflicts generally occur in poorer parts of the world, since it is more likely that poor people will

4 Interestingly Pierre and Sandrine (2011), show that pro government districts received lesser public investment than the pro opposition districts.
have a higher demand for the government services (see Bhattacharya et al. 2016 for a relevant discussion).

In comparison with the existing literature, the present paper fits in with the contribution of the role of government’s performance while political regime change is a contingent outcome. The issue of political outcome and government performance is not completely new. Greasley et al. (2011), Pacek (1994), Stokes et al. (1958), etc. examine various aspects of government performances vis-a-vis political outcomes from different part of the world. However, though it has been discussed extensively in the literature of political science, economists are not very categorical about this issue. Fairly large amount of literature exists on the side of political economy and public good provisioning separately. Our model is, somewhat, related to both these strands of literature. Although, instead of incorporating a “public good”, we designate the government providing “services” to the citizens. This is because states in reality seldom provide pure public goods to its citizens and the services they provide are often characterized by excludability and rivalry. A large number of paper deals in public economics with the question of “publicness” of public good, or more generally, public service (Haque 2001, Pesch 2008, Miller and Multon 2014 etc.). In fact, this property of excludability can become the source of political contention. Bowen (1943) and Stiglitz (1988) show that inefficient allocation of public good can arise in an equilibrium, when the outcome desired by the median voter differs from that of the standard optimum criteria envisaged by Samuelson. In Chari et al. (1997), bargaining between the presidential and local representative leads to an excessive number of public projects to be undertaken in the equilibrium. Persson et al. (1997) compares congressional and parliamentary political systems and shows that if the politician can appropriate money from the public good project for his/her own district, then once more a common pool problem may arise. Lizeri and Perisco (2001) compares a winner-take-all system (where all the spoils go to the winner) to a proportional system (where the spoils of office are split among the candidates proportionally to their share of the vote). It is shown that, in the former system, public good provided is lesser than the latter.

In a seminal contribution, Besley and Persson (2010) develop a model that analyses the incentives for states to invest in legal and fiscal capacities and relates this with the risk of external or internal conflict, the degree of political instability, and dependence on natural resources. This model is closely related to Besley and Persson (2010) in terms of its structure. Specifically, the government service is assumed to be a non-excludable consumption item, received directly from the state. Consumption of the agents also has a private component which is financed through the net (after tax) wage income. The indirect utility of the individuals thus depends directly on the quantity of government service available in the society (and their disposable labour income). The major departure from the model developed by Besley
and Persson (2010) is that we assume the probability of remaining in office for the incumbent leader depends positively on the level of government services provided. This has obvious implications for the intuitions derived in this set-up. Besley and Persson (2010) reports: “There now exists a large literature on conflict in the third world (see, e.g., Sambanis 2002 and Blattman and Miguel 2009 for broad reviews). Counting all countries and years since 1950, the incidence of civil war is about 6%, with a yearly peak of more than 12% (in 1991 and 1992), according to the Correlates of War data set. The cumulated death toll in civil conflicts since the Second World War exceeds 15 million (Lacina and Gleditsch 2005). A robust empirical fact is that poor countries are disproportionately more likely to be involved in civil war. There are two leading interpretations of this correlation in the literature: Fearon and Laitin (2003) see conflict in poor countries as reflecting limited capacity to put down rebellions by weak states, while Collier and Hoeffler (2004) see it as reflecting lower opportunity costs of fighting.”

Arguing that the literature on the civil war treats state capacity as exogenous, Besley and Persson (2010) develop a dynamic model of state capacity to explain the persistence of conflicts in the poorer parts of the world. We derive a similar result, but through a completely different channel. As discussed above, the performance of the government is gauged by the individuals as its ability to provide the service, at least this is what the incumbent leader perceives. Lower demand for government service entails that it is provided in a lesser volume, and this low demand implies that the opposition also has a reduced motivation to start a revolution. Moreover, the threat of armed conflict does not increase the provisioning of government service. Indeed regions with prolonged threat of armed coercion did not necessarily have better government services.

Section 2 develops the basic model without arms. Section 3 extends the basic model to incorporate the possibility of armed coercion. Finally, Section 4 concludes the model.

2 The model

In this section, we develop two consecutive models, the first one with agents who are “partially rational” and the second one with “fully rational” agents. In the first model developed in Subsection 2.1 and 2.2, the incumbent leader believes that the voters decide to vote, only on the basis of the public service provided. This case is restrictive, but has the advantage of being analytically tractable and also helps in understanding the solution technique adopted in the rest of the paper. Moreover, there is a debate in the literature on political economy about the effectiveness of taxation on electoral outcome. In Subsection 2.3, we build the second model with fully rational agent, where agents not only include taxes in their voting decision

---

5 See Stokes et al. (1958) for a discussion on how voters can have very different perspectives while deciding to exercise their franchise.

http://www.rei.unipg.it/rei/article/view/126
but also consider whether the stateless society provides or not more benefit.

2.1 Assumptions and structure with “partially rational agents”

The economy comprises of two distinct groups, namely the “incumbent group” and the “opposition-group”. The total population size is normalized to unity and is distributed uniformly within the interval $[0, 1]$. Without any loss of generality, the first half of the population distribution from $[0, \frac{1}{2}]$ is designated to be the incumbent group, while the distribution from $[\frac{1}{2}, 1]$ is that of the opposition group. To fix ideas, it is further assumed that there is no inter-group movement of agents. The two extremes of the distribution are inhabited by the two “leaders” and they are the decision-makers for their respective groups. There are two time periods, $s = 1, 2$; in the first one the incumbent leader is the ruler of the country. Before the second period begins (i.e. at the junction of the first and second period), there can be a change in power and the opposition leader may become the new ruler. All taxes are levied in the first period. Building closely on Besley and Persson (2010), it is assumed that the ruler can impose a discriminating taxation system, i.e. two different tax rates on the two groups. It is further assumed that these tax rates have both an upper bound and lower bound. Specifically, $t^j \in [-t, t]$ when $0 < t < 1$ and $j = \{O, I\}$. $O$ represents the opposition and $I$ represents the incumbent group. $t (-t)$ represents the fiscal infrastructure, or the constitutional restriction on the maximum possible tax (subsidy rate). To keep matters interesting, it is assumed that the state cannot siphon off the entire income of the agents ($t < 1$).

Tax receipts collected by the government are used to finance a particular government service $G$. The good $G$ is only partially non-excludable. The agents who receive this good (i.e., the part of the total population to whom the incumbent ruler bestows upon this service) shares this good in a non-rival and non-excludable fashion. However, it is indeed possible for the state to exclude some members of total population from the consumption of $G$. Thus, the total amount of government service provided in the economy

---

6 The two groups can be seen as two ethnic groups like Hindus and Muslims, or Tutshi and Hutu.

7 The dynamics of this power change is discussed later.

8 By subsidy, we mean negative values for $t^j$. We represent taxes by $t^j \in [-t, t]$. When $t^j \in [-t, 0)$, then the $j$-th individual receives a subsidy. Again when $t^j \in [0, t]$, then taxes are imposed strictly reducing the $j$-th individual’s income.

9 Actually it represents the aggregate public expenditure made by the government on the service. See the related discussion in Besley and Persson (2010).

10 Government announces budget allocation for certain project, like road, electrification etc. Since, as argued above, implementation actually takes place at the time of the next government (i.e., at second period), it can be suitably implemented. For example, if the eastern part of a country is strong political base of opposition and the western part is for incumbent of period 1, and if after the electoral process opposition of period 1 comes in power at period 2 then the leader may start to implement the public policy from the
is given by
\[ G = \frac{t^I}{2} + \frac{t^O}{2}. \] (1)

It should be noted, that though taxes are levied on the entire population according to their affiliations, the good \( G \) is not received by all. This follows from the fact that \( t < 1. \) Moreover, the incumbent leader would always enjoy the government service himself/herself, i.e., the allocation of \( G \) in this economy is such that it cannot preclude the incumbent leader from its ownership.\(^{11}\)

Besides the production of the service, the economy produces a final good (for consumption) using labour. Each agent is endowed with one unit of labour, which he/she supplies inelastically at the prevailing wage rate, \( w. \) The consumption good is produced using a constant returns to scale production function, and sold in a perfectly competitive market. Thus, the consumption good price is equalized with the wage. The utility function is linear in consumption, which in turn implies that the indirect utility depends on the net income (i.e. disposable income which is the income left to the agents after they have paid taxes). Indirect utility at any period, for a particular individual \( j, \) is given by
\[ U^j = \alpha(G)(D^j) + (1 - t^j)w = \alpha GD^j + (1 - t^j) \] (2)

where \( D^j = 0, \) if the \( j \)-th agent does not receive \( G \) and \( D^j = 1, \) otherwise. The second equality follows by setting \( w = 1, \) that is we choose labour to be the numeraire. It should be noted that the \( G \) feeds into the indirect utility function of the agents directly, and in a non-excludable and non-taxable manner. Following Besley and Persson (2010), \( \alpha \) is the valuation of the government service by the agents. Alternatively, one can consider it as a proportion of the total service provided, that becomes productive and enters into the budget constraints of individuals in a non taxable manner.\(^{12}\)

The incumbent leader believes that with probability \( \gamma, \) where \( \gamma \in (0, 1), \) it can retain its power in the next period. Moreover, the incumbent believes that this probability depends upon the amount of \( G \) it provides to the agents. It is for this reason that we term such agents as “partially rational”. Incumbent leader believes that agents care only about the amount of \( G \) received and not the opportunity cost of this service. Thus, without any loss of generality, it is assumed
\[ \gamma = G. \] (3)
Figure 1 - Service provisioning by the government

Figure 1 illustrates the above discussion. The line $AB$ represents the total population of unit mass and the two extreme points $A$ and $B$ are the positions of the incumbent and the opposition leaders respectively. Suppose in the first period the incumbent provides $G$ for all the people (who may belong to either of the groups) inhabiting the length $AC$. It would expect that all people residing in this region would support the incumbent leader in the next period (or rather at the junction of the two periods, when the power actually changes). In other words, the total amount of people the incumbent leader believes would support the ruling party is simply $G$. Since the construction of the model implies that $G \in [0,1)$ (see equation (1) and that $t < 1$), it is assumed that the amount of government service provided can be considered the measure of $\gamma$. Moreover, the structure of the model implies that the opposition leader never receives the service provided by the incumbent leader. This follows from the fact that the government cannot impose a tax equal to unity and thus from equation (1), $G$ is always less than one (the length of $AB$). In other words, government cannot provide the service to the entire population of length one, given the limits on the fiscal capacity ($t < 1$).

At the end of period 1, the incumbent leader announces the tax rate and at the juncture of two periods, the power can be transferred from incumbent to the opposition leader. The opposition leader now becomes the new incumbent ruler and faces the tax rate set for the incumbent by the previous government. The government service is provided in the second period. Note that taxes are announced on the basis of political positions, one for the incumbent and one for the opposition. Still, it remains binding in the

---

13 Note that assuming $\gamma = f(G)$ and $f'(G) > 0$ would have generated a qualitatively similar result. However, the specific form of equation (3) makes the model simpler and we have the result that no government service will be provided at all if its demand is sufficiently low.

14 Note that though in theory equation (1) does not rule out the possibility that the measure of the service provided by the government can be negative ($G < 0$), we simply assume that the Constitution of the country does not allow such a possibility to rise. Hence it is not feasible.
second period (when the service is actually received), while the identity of these two groups may actually change (with a probability \(1 - \gamma\)), i.e. the opposition may now become the ruler. Thus, while announcing the tax rates the incumbent leader would maximize a weighted sum of the gains that it would derive from remaining in power, and if it loses power and becomes the opposition. If there is no change in power, then the present incumbent leader would get a utility given by equation (2), while if it loses power and becomes the opposition leader, since there would be no access to the government service, would derive utility only from consumption. These cases can arise with the probabilities \(\gamma\) and \(1 - \gamma\), respectively. The expected utility of the incumbent leader is represented by \(\mathcal{E} V_{IR}^I\). Using equations (1), (2) and (3), \(\mathcal{E} V_{IR}^I\) can be expressed as:

\[
\mathcal{E} V_{IR}^I = \left[ (1 - t_I) + \frac{\alpha}{2} \left( t_I + t_O \right) \right] \left( \frac{t_I + t_O}{2} \right) + (1 - t_O)(1 - \frac{t_I + t_O}{2}). \tag{4}
\]

Though \(\mathcal{E} V_{IR}^I\) is the expected utility of the incumbent leader, still the negative impact for taxes for both the groups enter in the \(\text{RHS}\) of equation (4). This is because the incumbent leader also takes into account the fact that in the next period, it may lose power and become the opposition. Thus a high tax rate on the opposition in that situation may become too costly in that case (see the second term in the \(\text{RHS}\) of equation (4). The opposition leader, in this basic model, does not have any policy variables under its control and thus has to passively face the tax rates imposed by the unilateral decision of the state.

### 2.2 Equilibrium with partially rational agents

Interestingly, if there is no chance of power change, then \(\mathcal{E} V_{IR}^I = 1 - t_I + \frac{\alpha}{2} (t_I + t_O)\), which implies that the incumbent leader would always set \(t_O^0 = t\) for the opposition, and for its own group members would set a tax rate \(t_I^* = t\) when \(\alpha > 2\) and \(t_I^* = -t\) if \(\alpha < 2\). A relatively high value of \(\alpha\) implies a high demand of the government service (compared to that for private consumption) and thus even when there is no threat of change in power the incumbent right wing leader would provide the maximum possible amount of \(G\) to both the groups. However, if the demand of state service is relatively low (\(\alpha < 2\)), then the right wing leader would provide the maximum subsidy possible to the own group members. Members of the opposition would have to pay the maximum tax in both these situations. In other words, for high \(\alpha\), the incumbent taxes both groups at the maximum fiscal capacity and spends all the revenue on \(G\). If state provided services are not very valuable as opposed to private consumption, no service would be provided and all available tax receipts are transferred to the incumbent group (through the maximum possible negative tax rate).\(^{15}\)

\(^{15}\)This result is similar to Besley and Perrson (2010).
Now, if a credible threat of power change does exist, and if the probability of power change depends on state service provisioning, then the objective function of the incumbent leader should take the utility of opposition leader into account. This implies that the right wing leader would set $t_I$ and $t_O$ to maximize equation (4). Differentiating the expected return of the ruler, with respect to the tax rates, the following expressions can be obtained:

$$\frac{\delta EV^R_I}{\delta t_O} = -1 + t^O + \frac{\alpha}{2} (t^I + t^O)$$  \hspace{1cm} (5)

and

$$\frac{\delta EV^I_R}{\delta t_I} = -t^I + \frac{\alpha}{2} (t^I + t^O).$$  \hspace{1cm} (6)

The first-order conditions for maximization (obtained by equating the RHS of the eqs. 5 and 6 to 0) are no longer valid in this case, as the second-order sufficiency condition is not satisfied. The basic intuition of the optimization technique, adopted throughout the rest of the paper, is separating out the objective function in different zones where the function is increasing according to different directional movements of the policy variables, namely, $t_I$ and $t_O$.\[16]

**Proposition 1** In the present model, if the demand for government services is high i.e. $\alpha > \frac{2}{\gamma}$, then $t^*_I = t^*_O = t$, while if there is a low demand for government service i.e. $\alpha \leq \frac{2}{\gamma}$ then $t^*_I = t$, $t^*_O = -t$.

**Proof.** See Appendix. \[\square\]

Proposition 1 is the central result of this section. If the demand for government service is high enough (reflected by a high $\alpha$, i.e. $\alpha > \frac{2}{\gamma}$), then the service will be provided in its full capacity (taxes are charged at the maximum). If the agents do not have much demand for the service (i.e. $\alpha \leq \frac{2}{\gamma}$), no government service will be provided at all. This directly follows from the optimization program of the leader. More interestingly, maximization of expected utility by the incumbent leader entails that in such a case the benefit of the subsidy will be provided to the opposition. This is because a low value of $G$ implies, that $\gamma$ is lower and the incumbent ruler strongly believes it will lose power. Thus, in the second period, it will be in the position of the opposition and thus announces subsidy for the opposition in the first period. Proposition 1 also implies that, for a given level of demand for government service, in those countries where fiscal infrastructure is not strong (low $t$), public services are less likely to be provided (since for provisioning of $G$, $\alpha$ has to be greater than $\frac{2}{\gamma}$).\[17]

\[16\] The relevant Hessian determinant does not alternate in sign, which is the sufficient second order condition for maximization.

\[17\] It should be noted that, since $\alpha$ is a constant, this is not a case of multiple equilibrium. Proposition 1, describes the various possible equilibriums for different permissible values of $\alpha$. For a given $\alpha$, we have an unique equilibrium.
2.3 Equilibrium with fully rational agents

In the model developed in the previous section, the belief of the state is that agents who inhabit the economy care only about the amount of government service received. The only reason that taxation causes dis-utility is due to the fact that it reduces disposable income, and thereby reduces consumption (see equation 2). As a consequence of such a belief, government expects the probability of getting again the power to be directly equal to the amount of service received by the people (see equation 3). In this section we develop a model where the state believes that people would vote for government party only if the net return received from the government service after taxes is positive. Formally, if the amount of government received by each agent is $G$ and the tax levied $t$, then:

$$\gamma = N(S),$$  \hspace{1cm} (7)

where $S$ is the set of people with $\alpha G - t > 0$, and $N(S)$ represents the cardinal number of the set $S$, i.e. those people who get net benefit from the government service after paying the tax. For pedagogical purposes we designate such agents as “fully rational”. Such agents are perceived by the government as evaluating the role of the state as an efficient provider of the service. The state can only hope to garner support of an individual $s$ if and only if $s \in S$. The equilibrium is characterized in three successive steps, where the valuation of the government service is very high ($\alpha > 2$), is moderate ($\alpha = 2$) and is very low ($\alpha < 2$) respectively. Notion of the equilibrium remains the same as earlier. Incumbent leader maximises its expected utility with respect to the taxes imposed.

The expected utility of incumbent leader is:

$$EV_{IR} = \left[ (1 - tI) + \alpha G \right] \gamma + (1 - tO)(1 - \gamma).$$  \hspace{1cm} (8)

From equation (1), this can be expressed as

$$EV_{IR} = 1 - tI(1 - \frac{\alpha}{2})\gamma - tO[1 - \gamma(1 + \frac{\alpha}{2})].$$  \hspace{1cm} (9)

Notice that the expected utility of the incumbent leader depends both on the taxes levied on the incumbent members ($tI$) and on the opposition members ($tO$). For different choices of these taxes, $\gamma$, would be different. That,
in turn, implies that we cannot optimise \( EV^I_R \) directly in terms of the tax rates. For different range of these tax rates, \( \gamma \) would be different. The nature of \( \gamma \) is quite different from the previous section. Here \( \gamma \) is not continuous and monotonic in terms of public expenditure and taxes. Thus, the optimisation programme of the incumbent leader becomes a bit involved. We need to separate out zones of \( t^I \) and \( t^O \), and determine \( \gamma \) in terms of the tax rates.\(^{21}\) For each of these ranges, the locally optimal strategy of the incumbent leader is identified. Comparing these local optima we can find the global optimal.

**Case I: Moderate valuation of state provided service (\( \alpha = 2 \))**

In this case,

\[
EV^I_R = 1 - t^0(1 - 2\gamma). \quad (10)
\]

(i) Suppose \( t^I = \beta^I t \) and \( t^O = \beta^O t \), and \( 0 \leq \beta^I, \beta^O \leq 1.\)\(^{22}\) So, the total amount of service available will be \( \frac{1}{2}(\beta^I + \beta^O)t \) and since all those who receive the government service would benefit, \( \gamma = G. \) Substituting this into equation (10), \( EV^I_R \) becomes an increasing function of \( \beta^I. \) Thus, it would be optimal to choose \( \beta^I = 1. \) Then

\[
EV^I_R = 1 - \beta^O t + \beta^O t^2(1 + \beta^O). \quad (11)
\]

As \( \beta^O \in [0, 1], EV^I_R \) is a convex function. Moreover, \( EV^I_R(\beta^O = 0) = 1, \) and \( EV^I_R(\beta^O = 1) = 1 - t + 2t^2. \) Hence,

\[
EV^I_R(\beta^O = 1) - EV^I_R(\beta^O = 0) = t(2t - 1).
\]

Thus, if \( t > \frac{1}{2}, \) \( \beta^I = \beta^O = 1 \) and \( EV^I_R = 1 - t + 2t^2. \) If \( t < \frac{1}{2}, \) then \( \beta^I = 1 \) and \( \beta^O = 0, \) while \( EV^I_R = 1. \) Else, \( t = \frac{1}{2} \) and the incumbent leader is indifferent between the two situations.

(ii) Suppose, \( t^I = \beta^I t, \) and \( t^O = -\beta^O t, \) where \( 0 < \beta^I, \beta^O \leq 1. \) Then, \( G = \frac{1}{2}(\beta^I - \beta^O)t < \frac{1}{2}. \) Clearly all opposition group members would support, but no member of the incumbent group would. Thus, \( \gamma^* = \frac{1}{2} \) and \( EV^I_R = 1 \) (see equation (11)). If \( t > \frac{1}{2}, \) (i) dominates (ii), and for \( t \leq \frac{1}{2}, \) both (i) and (ii) are equivalent.

(iii) Suppose, \( t^I = -\beta^I t, \) and \( t^O = \beta^O t \) and \( 0 < \beta^I, \beta^O \leq 1. \) Then \( G = \frac{1}{2}(\beta^I - \beta^O)t < \frac{1}{2}. \) It is easy to check that opposition members would not support, but all incumbent members would support. Thus, \( \gamma^* = \frac{1}{2} \) and \( EV^I_R = 1 \) (see equation (11)). The rest of the analysis is similar to the previous case.

The discussion from (i) to (iii) can be summarized in the following proposition:

---

\(^{21}\) In each of these zones \( \gamma \) is continuous.

\(^{22}\) Introduction of \( \beta^O \) and \( \beta^I \) are for notational ease.
Proposition 2 In the present model with fully rational agents, if the demand for government service is moderate ($\alpha = 2$), and if $t > \frac{1}{2}$, then $t^I = t^O = t$. Otherwise, there are multiple equilibrium.

Proposition 2 says that when the valuation for the government service is moderate, then the state would provide the service only if the fiscal capacity is high enough. On the other hand, if the state’s fiscal capacity is low, then in case of moderate valuation of $G$, the role of the state becomes passive. Relocation through tax and subsidy does not give any unique global optimum.

Case II: High valuation of state provided service ($\alpha > 2$)

In this case also, equation (9) gives the return of the incumbent leader. We proceed to establish the equilibrium, following the same procedure as in the previous case.

(i) Suppose $t^I = -\beta^I t$, and $t^O = \beta^O t$, where both $\beta^I$ and $\beta^O$ are strictly positive, less than unity and $\beta^O > \beta^I$. The total amount of government service available with the economy will be $G = \frac{1}{2}(\beta^O - \beta^I)t$, which is less than $\frac{1}{2}$. All right wing members, those who get the service and those who do not get would support (since those who do not receive the government service will get a subsidy). Contrarily, the opposition members would not support. Thus, the incumbent leader would expect that half of the population would support it during regime change, and thus $\gamma = \frac{1}{2}$. From equation (9), the utility of the incumbent leader becomes

$$EV^I_R = 1 - \frac{\beta^I t (\alpha/2 - 1)}{2} + \frac{\beta^O t}{2} \left( 1 + \frac{\alpha}{2} \right).$$

(12)

Optimality demands that the incumbent leader chooses $\beta^O = 1$ and $\beta^I = \epsilon$, where $\epsilon$ is a very small positive number. Thus, the expression in equation (12) can be expressed as

$$EV^I_R = 1 - \frac{\epsilon t}{2} \left( \frac{\alpha}{2} - 1 \right) + \frac{t}{2} \left( \frac{\alpha}{2} - 1 \right).$$

(13)

(ii) Suppose, $t^I = 0$ and $t^O = \beta^O t$. Then some incumbent members get $G = \frac{\beta^O t}{2}$, and do not have to pay any taxes, while some neither receive the government service or pay taxes. On the other hand, the opposition pays taxes, but does not receive any government service (as $G < \frac{1}{2}$ in this case). Thus $\gamma = \frac{\beta^O t}{2}$. Using equation (9), it is straightforward to check that the utility of the incumbent leader in this case is:

$$EV^I_R = 1 - \beta^O t + \frac{\beta^O t^2}{2} \left( 1 + \frac{\alpha}{2} \right).$$

(14)
which is a convex function of $\beta^O$. The incumbent leader would either choose $\beta^O = 0$ which yields an utility $EV^I_R(0, 0) = 1$, or $\beta^O = 1$ with corresponding utility $EV^I_R(0, 1) = 1 - t + \frac{t^2}{2}(1 + \frac{\alpha}{2^2})$, if $t < \frac{4}{(\alpha+2)}$, or $t > \frac{4}{(\alpha+2)}$, respectively.

(iii) Now consider the polar opposite case. That is, the incumbent taxes only its own group members. So $t^I = \beta^It$, where again $\beta^I$ is strictly positive and less than unity, and $t^O = 0$. Following a similar argument as in case (ii), $\gamma = \frac{\beta^I}{2}$ and the expected utility of the incumbent leader would be $EV^I_R(1, 0) = 1 + \frac{t^2}{2}(\frac{\alpha}{2} - 1)$.

(iv) Suppose $t^I = \beta^It$ and $t^O = -\beta^O t$, where both $\beta^I$ and $\beta^O$ are strictly positive and $\beta^I > \beta^O$. Then, the amount of government service available in the economy would be $G = \frac{(\beta^I - \beta^O)t}{2}$. All the opposition members would support the government, while only those incumbent members who receive the government service would support. Thus, the government would expect that $\gamma = \frac{(\beta^I - \beta^O)t}{2} + \frac{1}{2}$. This gives an expected utility $EV^I_R = 1 + (\frac{\alpha}{2} - 1)\frac{1}{2}(\beta^I - \beta^O) + \frac{t^2}{2}(\frac{\beta^I - \beta^O}{2} - (\beta^I - \beta^O)^2)$. The incumbent leader would choose $\beta^I = 1$, and $\beta^O = -\epsilon$, where $\epsilon$ is a very small positive number.

(v) Finally, let us assume that the incumbent leader taxes both the groups in the economy. So $t^I = \beta^It$, $t^O = \beta^O t$ and both $\beta^I$ and $\beta^O$ are strictly positive. Since $\alpha > 2$, for all those who receives the service, $\alpha G - t^I$ is positive, where $k \in (I, O)$. Hence, $\gamma = G = \frac{(\beta^I + \beta^O)t}{2}$. The incumbent leader would choose, $\beta^I = \beta^O = 1$ if $\alpha > \frac{4-2\epsilon}{3\epsilon}$ and would receive an utility $EV^I_R(1, 1) = 1 + \alpha^2 - t$. Otherwise, the government would set $\beta^I = 1$, and $\beta^O = 0$, which gives utility $EV^I_R(1, 0) = 1 + \frac{t^2}{2}(\frac{\alpha}{2} - 1)$.

Our analysis from cases (i) to (v), when $\alpha > 2$, implies that the expected utility of the incumbent leader is simply a function of $\beta^I$ and $\beta^O$, i.e. $EV^I_R = EV^I_R(\beta^I, \beta^O)$. The optimal strategy of the government would be to compare the expected utility levels in each case and then choose $\beta^I$ and $\beta^O$ to obtain the maximum utility level possible. First let us compare $EV^I_R(-\epsilon, 1)$ and $EV^I_R(0, 1)$. See equation (11) and case (iii).

$$EV^I_R(-\epsilon, 1) - EV^I_R(0, 1) = \frac{\alpha t}{4}(1 + t) + \frac{t}{2}(1 - t) - \epsilon(...) > 0 \forall \alpha,$$

when $\epsilon$ is sufficiently small. Moreover, for a relatively higher fiscal capacity, $t > \frac{1}{2}$, $EV^I_R(-\epsilon, 1) > EV^I_R(1, 1) \frac{(t - 1)}{1 - 3t}$ and $EV^I_R(-\epsilon, 1) < EV^I_R(1, 1)$, otherwise.

When fiscal capacity is low, $t < \frac{1}{2}$, then case (iv) dominates all the other possible situations. The expected utility is thus $EV^I_R(1, -\epsilon)$. (See the discussion of case (iv).) Proceeding in this way, we can rank all the possible

---

23 In this case, it can be shown that $EV^I_R(1, 0) > EV^I_R(0, 0)$ for all $\alpha > 2$ and $t < 1$.

24 $EV^I_R(-\epsilon, 1) - EV^I_R(1, 1) = \frac{1}{2}(1 + \frac{\alpha}{2}) - \alpha^2$, which follows from cases (i) and (v). The above result is obtained from this difference along with the fact that $\alpha > 2$.

25 $EV^I_R(-\epsilon, 1) \leq EV^I_R(1, -\epsilon)$ when $t < \frac{1}{2}$.
Proposition 3 In the present model with fully rational agents, if the demand for state provided service is high ($\alpha > 2$), and if $t > \frac{1}{2}$, then $t^I = t^O = t$ if and only if $2 < \alpha < \frac{2(t-1)}{1-3t}$. If $t > \frac{1}{2}$, and $\alpha > \frac{2(t-1)}{1-3t}$, then there exists no equilibrium. Also if $t < \frac{1}{2}$, then no equilibrium exists.

Proof. For $t > \frac{1}{2}$, and $2 < \alpha < \frac{2(t-1)}{1-3t}$, it has already been shown that $EV_R^I(1,1) > EV_R^I(\epsilon,1)$. Also from the above discussion, $EV_R^I(\epsilon,1) > EV_R^I(0,1)$ $\forall \alpha$. Moreover from cases (ii) and (v), $EV_R^I(1,1) > EV_R^I(1,0)$, which proves the first part of the proposition. If the fiscal capacity is relatively low, $t < \frac{1}{2}$, then $EV_R^I(1,-\epsilon)$ dominates all possible situations. Since for any choice of $\epsilon$, there exists a still smaller number for which the expected utility is maximized, clearly there is no equilibrium in this case.

Note that $\frac{2(t-1)}{1-3t}$ should be greater than 2, and that happens only if $t > \frac{1}{2}$. So a higher fiscal capacity coupled with moderately high valuation of the state provided service gives incentive to provide the highest amount of the service. On the one hand, with a very high value of government service, it becomes optimal for the incumbent leader to provide subsidy to its own group members. In this case, the increase in the vote share and in the private benefit to the leader outweighs the gain from more government service. For lesser fiscal capacity, incumbent leader tries to increase the probability of retaining power by subsidizing the opposition. On the other hand, if the incumbent leader loses, it would benefit from the subsidy.

Case III: Low valuation of state provided service ($\alpha < 2$)

(i) Suppose $t^I = \beta^It$, $t^O = -\beta^Ot$ and $\beta^I > \beta^O > 0$. That is the incumbent leader taxes its own group members, while opposition members receives subsidy. Total amount of government service in the economy is $G = \frac{1}{2}(\beta^I - \beta^O)t$. In this situation the incumbent group members would not support, while those in opposition would support the incumbent leader. Thus, $\gamma = \frac{1}{2}$. Substituting this into equation (9), the expected utility of the incumbent leader is:

$$EV_R^I = 1 - \frac{1}{2}(\beta^I - \beta^O)t \leq 1.$$ (16)

The incumbent leader maximises the expected utility, by choosing $\beta^I = \beta^O = \beta^*$, and correspondingly $EV_R^I = 1$.

(ii) Consider the polar opposite case, i.e., the incumbent leader taxes the opponent group, while subsidises its own members. Thus, $t^I = -\beta^It$ and $t^O = \beta^Ot$ and $\beta^O > \beta^I > 0$. This is very similar to the previous case, and $\gamma = \frac{1}{2}$, while $EV_R^I = 1 - \frac{1}{2}(\beta^O - \beta^I)t \leq 1$. The optimal choice and
the expected utility level of the incumbent leader would be similar to the previous case.

(iii) Suppose \( t^I = 0 \) and \( t^O = \beta^O t \). This in turn implies that only some of the incumbent members would get the government service, and the total amount of government service available in the economy would be \( G = \frac{1}{2} \beta^O t \). Since all those who receive the government service actually has to pay taxes, \( \gamma = \frac{1}{2} \beta^O t \). Thus, \( EV^I_R = 1 - \beta^O t + \frac{\beta^{O2} t^2}{2} (1 + \frac{\alpha}{2}) \), which is a convex function and \( EV^I_R(\beta^I = 0, \beta^O = 0) = 1 > EV^I_R(\beta^I = 0, \beta^O = 1) = t + \frac{t^2}{2} (1 + \frac{\alpha}{2}) \).

(iv) Consider the case, that \( t^I = \beta^I t \) and \( t^O = 0 \). In this case, nobody would support the incumbent leader and as a result \( \gamma = 0 \), and \( EV^I_R = 0 \).

(v) Suppose that the incumbent leader imposes tax on both groups. That is \( t^I = \beta^I t \) and \( t^O = \beta^O t \), where both \( \beta^I, \beta^O \geq 0 \). Then total amount of government service in the economy is \( G = t^2 (\beta^I + \beta^O) \). Now the probability that the incumbent leader would retain power can be either greater, lesser or equal to \( G^{26} \). Since \( \alpha < 2 \), this situation is arising in case (v). For the different possible positive combinations of the tax rates, corresponding support base of the incumbent leader would be different. For example, if \( G > \frac{1}{2} t \) and all those who receive the government service is better off due to redistribution then \( \gamma = G \). However, if only opposition members who receive the government service is benefited then \( \gamma = G - \frac{1}{2} t \). It can be shown that for \( \gamma = G \), the expected utility would always be higher.\(^{27}\) Similarly, there are other sub cases also. Thus in this case, we need to segregate the objective function of the incumbent leader into appropriate zones, and then find the global optimum from the different local optimum from each zone. Thus, we need to concentrate only on this case. From, equation (9), the expected utility of the incumbent leader is:

\[
EV^I_R = 1 - \beta^I t (1 - \frac{\alpha}{2}) t^2 (\beta^I + \beta^O) - \beta^O t [1 - (1 + \frac{\alpha}{2}) \frac{t}{2} (\beta^I + \beta^O)].
\]  

(17)

It can be shown that the expression in equation (17) does not have an interior solution, when it is maximised in terms of \( \beta^I \) and \( \beta^O \).

It is sufficient to evaluate equation (17), only at the corner points, i.e. when either of the taxes imposed are zero, or both are equal to \( t \), or one of them is \( t \), while the other is zero.

\[
EV^I_R(\beta^I = 0, \beta^O = 0) = 1 \quad (18)
\]

\[
EV^I_R(\beta^I = 1, \beta^O = 0) = 1 + \frac{t^2}{2} (1 + \frac{\alpha}{2} - t) \quad (19)
\]

\(^{26}\) This is because \( \frac{1}{2} (\beta^k - 1) + \frac{1}{2} (\beta^v) \) is of ambiguous sign, where \( k \) and \( v \) represents two different groups.

\(^{27}\) The proof of this statement is available from the authors on request.


\[ EV_I^{L}(\beta^I = 0, \beta^O = 1) = 1 - \frac{t^2}{2} \left(1 - \frac{\alpha}{2}\right) \]  

(20)

\[ EV_I^{L}(\beta^I = 1, \beta^O = 1) = 1 + t^2 \alpha - t. \]  

(21)

Also, \( EV_I^{L}(\beta^I = 1, \beta^O = 1) = (1 + t^2 \alpha - t) > 1 = EV_I^{L}(\beta^I = \beta^O = 0), \) if \( \alpha > \frac{1}{t}. \)

The above discussions directly lead us to the following proposition.

**Proposition 4** In the present model with fully rational agents, when the valuation of the government service is low, that is \( \alpha < \frac{2}{t} \), then if \( \alpha > \frac{1}{t} \), then \( t^I = t^O = t \). If \( \alpha < \frac{1}{t} \), then \( t^I = t^O = 0 \).

The above proposition says that when the demand for the state provided services is low then, the chance of provisioning the service is inversely related with the fiscal policy for a given value of \( \alpha \). That is, for a given value of \( \alpha \), which is less than 2, if the fiscal capacity is low enough, then it is highly probable that government will not provide any service in the equilibrium (for example, when \( t < \frac{1}{2} \)).

### 3 Discussion: Active opposition

The previous two sections contain detailed analysis of the possible strategies of the incumbent leader. The role of the opposition is very passive, if not silent. One can interpret the stochastic nature of the political regime as a passive role of the opposition. That is, the probability of not retaining power by incumbent leader is non zero (strictly positive). Therefore \( 1 - \gamma \) can be considered as an index of the performance of opposition. However, this can also be explained as nothing but a manifestation of the vote against the incapability of the ruling government (almost equivalent to anti-incumbency force). This model can thus encompass a simple extension where the opposition is active and can potentially invest to reduce the probability of retaining power by the ruling government.

For notational ease (and to give a realistic idea), costly action of the opposition leader will be termed as “investment in arms” (or just as “armed conflict”). The investment in arms by the opposition leader does not necessarily imply a successful transfer of political authority. Nonetheless, strictly positive investment in arms by the opposition leader will reduce the probability of winning for the incumbent leader (say, \( \gamma^A \) is now the probability of winning for the incumbent leader). A convenient form is the following:

\[ \gamma^A = G - \mu A^O. \]  

(22)

where \( 0 < \mu < 1 \) and is a measure of the marginal impact of arms, developed by the opposition, on the probability of retaining power by the incumbent. \( A^O \in [0, A] \) is the amount of arms the opposition decides to develop. It is further assumed that the amount of arms that can be developed by
the opposition leader can at the most be $A$, which is bounded from above by the fiscal capacity of the state i.e. $A < t$. The basic difference between the models developed in this section and the model developed in the previous section is that while in the previous section the left wing opposition leader was a passive recipient, here it has one strategic variable at its disposal by virtue of which it can influence the probability of power transfer. The remaining characterization of the equilibrium for both partially rational agents and fully rational agents is similar. We will explain the technique and the result for the basic model with partially rational agents, in brief.

The new objective function of the incumbent leader will be very similar to the previous cases:

$$EV_{RL} = \left[ (1-tI) + \frac{\alpha}{2}(tI+tO) \right] \left( \frac{tI + tO}{2} - \mu A^O \right) + (1-tO)(1-tI)(1-tI + tO) + \mu A^O. \quad (23)$$

Now, the optimization exercise for the incumbent leader is a two stage optimization problem, because $A^O$ is an endogenous variable but is determined by the opposition leader. In the first stage incumbent leader follows the technique of optimization as in section 2, while considering $A^O$ as given. Therefore this optimisation gives the best response function to the incumbent leader for each value of $A^O$.

The opposition leader will invest in arms only if the gain from investing is strictly positive in comparison with not investing in arms. Opposition leader considers the tax rates announced by the incumbent leader as given. Suppose the payoff of the opposition is $EV_{L}(A^O \neq 0)$ when the opposition invests and produces arms, and $EV_{L}(A^O = 0)$ be the payoff when the opposition leader decides not to invest in arms. Thus, if $\phi(= EV_{L}(A^O \neq 0) - EV_{L}(A^O = 0)) > 0$, then $A^O > 0$, and it is contingent upon the tax rates set by the incumbent leader. Therefore, the best response function of the opposition leader for each action of the incumbent leader can be obtained in this way. Finally, the equilibrium can be obtained from the two best response functions. The main result of this simple extension of the basic model is given by the following proposition:

**Proposition 5**

i) If $\alpha > \frac{2(1+\mu A)}{t-\mu A}$ and $\alpha t > \frac{1}{\mu}$, then $t^*_I = t^*_O = t$ and $A^O_\ast = A$.

ii) If $\alpha > \frac{2}{t}$ and $\mu t < \frac{1}{\mu}$, then $t^*_I = t^*_O = t$ and $A^O_\ast = 0$.

iii) If $\alpha < \frac{2}{t}$ then $t^*_I = t$, $t^*_O = -t$ and $A^O_\ast = 0$.

We skip the proof of the above proposition.\(^{28}\)

In case the valuation of the government service is less (low level of $\alpha$), opposition is going to get the subsidy. The intuition is similar to that in

\(^{28}\) The proof is available on request from the authors.
the previous section. Moreover, to have a situation of armed conflict, $\alpha$ should be sufficiently high along with a high marginal impact of arms on the probability of power change ($\mu$). Therefore, armed conflict does not work as a threat in state service provisioning in this model. In other words, if it is more likely, that the service will be provided by the government, then only it will be incentive compatible (such that it outweighs the cost of arm accumulation) for the opposition leader to invest in arms to swing the power in his/her favour. Otherwise, it is not beneficial for the opposition leader to fetch the power in the next period by costly means. Therefore, the opposition leader will become proactive only in those cases where the marginal benefit from the government service is very high and not the other way round, that is, it does not push the government to provide the service.

One can carry out a similar exercise for the fully rational agents. The techniques and results are grossly similar and intuitive. The motive of the above discussion is to show that this very simple model is capable to incorporate an active opposition leader.

4 Conclusion

Welfare states typically provide various services for its citizens. Another defining feature of modern welfare states is intense political rivalry among various heterogeneous group. This paper has considered the interaction between these two factors. Interestingly, the model had emphasized the causal direction from fiscal policy to political regime change. We have studied the situation in which the decision of the voters are believed to be affected by the supply of public services by the government. The model has considered two types of agents: partially rational agents and fully rational agents. The former values only the provisioning of the government service, while the latter compares the gain from the government service vis-a-vis no redistribution. With fully rational agents, in some cases no equilibrium may exist, though optimal policy direction can be obtained. It has been shown that low demand for government service can lead the incumbent leader to actually subsidize the opposition. Moreover, as an extension, the scope to model an active opposition and the possible corresponding results have been discussed. We argue that the opposition leader would only have an incentive to foster an armed revolution (that is typically costly action) when the demand for the state provided services is sufficiently high. Therefore, costly armed action may not become a threat to the provisioning of government services.

29 This extension for fully rational agents is available on request.
Appendix

This appendix provides the formal proof of Proposition 1. To arrive at this proposition, we proceed through the following lemmas.

Equations (5) and (6) are plotted in Figure 2 in the \((t^I, t^O)\) plane when \(\alpha < 2\). These lines divide the plane into four zones designated as \((i), (ii), (iii)\) and \((iv)\) and lines \(PQ\) and \(RS\) represent \(\frac{\delta EV_I^R}{\delta t^O} = 0\) and \(\frac{\delta EV_I^R}{\delta t^I} = 0\) respectively.

**Figure 2 - Optimal choice of tax rates in absence of arms when \(\alpha < 2\)**

![Figure 2](image)

**Lemma 1** Suppose \(EV_I^R(l)\) represent the maximum possible expected utility of the incumbent right wing leader, in the zone \(l\), where \(l \in \{(i), (ii), (iii), (iv)\}\) and the ruler makes the optimal choice within a particular zone. Then, when \(\alpha < 2\)

\[
EV_I^R(i) = 1 + t + \alpha t^2,
\]

\[
EV_I^R(ii) = 1 - t,
\]

\[
EV_I^R(iii) = 1 - t(1 - \alpha t),
\]

and

\[
EV_I^R(iv) = 1 + t.
\]

**Proof.** Consider the line \(PQ\), above this line for any given value of \(t^I\), \(\frac{\delta EV_I^R}{\delta t^O} > 0\) (this follows directly from equation 5). So it is always optimal for the incumbent leader to choose, \(t^O = t\) in this region (\(\forall t^I\)). Below the line \(PQ\), for any given value of \(t^I\), \(\frac{\delta EV_I^R}{\delta t^O} < 0\). So it is always optimal for the
ruler to choose \( t_s^O = -t \) (\( \forall t^I \)).

Similarly, consider the line \( RS \). Above this line, given a particular value of \( t^O \), \( \frac{\partial EV_R^I}{\partial t^O} < 0 \) (this follows directly from equation 6). The incumbent leader should set \( t_s^I = -t \) (\( \forall t^O \)). Below this line \( RS \), \( \frac{\partial EV_R^I}{\partial t^I} > 0 \). The optimal choice for the ruler is \( t_s^I = t \) (\( \forall t^O \)).

Therefore from Figure 2, this would imply that in

- zone (i), \( t_s^O = -t \) and \( t_s^I = -t \),
- zone (ii), \( t_s^O = t \) and \( t_s^I = -t \),
- zone (iii), \( t_s^O = t \) and \( t_s^I = t \),
- zone (iv), \( t_s^O = -t \) and \( t_s^I = t \).

Substituting these values into equation (4), the lemma is proved.

Assuming that \( \alpha > 2 \), equations (5) and (6) are plotted in Figure 3 in the \((t^I, t^O)\) plane. It divides the plane into four zones designated as (v), (vi), (vii) and (viii) and lines \( EF \) and \( GH \) represent \( \frac{\partial EV_R^I}{\partial t^O} = 0 \) and \( \frac{\partial EV_R^I}{\partial t^I} = 0 \), respectively.

**Figure 3 - Optimal choice of tax rates in absence of arms when \( \alpha > 2 \)**

**Lemma 2** Suppose \( EV_R^I(j) \) represents the maximum possible expected
utility of the incumbent right wing leader, in the zone \( j \), where \( j \in \{ (v), (vi), (vii) \text{ and } (viii) \} \) and the ruler makes the optimal choice within a particular zone. Then, when \( \alpha > 2 \),
\[
EV_R^I(v) = 1 + t + \alpha t^2, \\
EV_R^I(vi) = 1 - t, \\
EV_R^I(vii) = 1 - t(1 - \alpha t) \text{ and} \\
EV_R^I(viii) = 1 + t.
\]

The proof is identical to that of Lemma 1. Interestingly, lemma 1 and 2 imply that regardless of the level of demand for \( G \), the entire choice space of the incumbent leader can be divided in terms of the symmetric optimal values of \( t^I \) and \( t^O \). Suppose \( a, b, c \) and \( d \) represent the zones where, the incumbent leader is constrained to operate. Then, \( EV_R^I(a) = 1 + t + \alpha t^2, \)
\[
EV_R^I(b) = 1 - t, EV_R^I(c) = 1 - t(1 - \alpha t) \text{ and } EV_R^I(d) = 1 + t. \]

Lemmas 1 and 2 can be combined to arrive at proposition 1. To facilitate our exposition, we provide a formal proof of proposition 1 below.

**Proof of proposition 1**

**Proof.** Clearly, \( EV_R^I(a) = 1 + t + \alpha t^2 \) is not possible. This is simply because it is not feasible (as in this case \( t^I = t^O = -t \), this would imply \( G < 0 \)). Also, \( EV_R^I(b) = 1 - t < EV_R^I(c) = 1 - t(1 - \alpha t) \) and thus can never be the equilibrium pay off of the leader. Comparing \( EV_R^I(c) \) and \( EV_R^I(d) \), it is found that:
\[
EV_R^I(c) > EV_R^I(d) \text{ if } \alpha > \frac{2}{7} \\
EV_R^I(c) \leq EV_R^I(d) \text{ if } \alpha \leq \frac{2}{7}.
\]

From lemmas 1 and 2, the optimal choices of the tax rates are \( t^I = t^O = t \) and \( t^I = t, t^O = -t \) when the expected returns of the incumbent leader are \( EV_R^I(c) \) and \( EV_R^I(d) \) respectively. Hence, the proposition is proved. \( \square \)

---

30 In Figure 1, \( a, b, c \) and \( d \) are represented as \( (i), (ii), (iii) \) and \( (iv) \) respectively. In Figure 2, they are represented as \( (v), (vi), (vii) \) and \( (viii) \) respectively.
References


http://www.rei.unipg.it/rei/article/view/126


