Appendix 1: Formal model of benefit maximization in a resource-dependent economy

Here we present the formal model. After first describing the benefits and costs of the resource and its governance, we define and solve the optimal control problem. We then examine the results in terms of three phases of growth: extensive growth, intensive growth and capitalization, and trade.

A1.1. A resource-dependent economic system

Society, having a laboring population level of n_{2t}, strives to maximize the value derivable from their resource base, n_{1t}, both today and into the future. This maximization process is modeled with a dynamic optimal control multi-level predator-prey model. Society maximizes economic value (total net benefits), over time, through costly decisions about harvest restrictions ($\gamma_t(I_t)$) on the resource as a function of institutional structure (I_t) and the shares of the resource to non-laborer consumption. These consist of the share ($s_t(I_t)$) of the resource to trade and the share $(\varphi_t(I_t))(1-s_t(I_t))$ of the resource (through labor) to elite formation and/or capital accumulation (K_{2t}). The remaining share of the resource, $(1-s_t)(1-\varphi_t)$, goes to laborer consumption.

A1.2 Value of the resource over time

The value of the resource to society depends upon its division: the share to immediate consumption $(1-s)(1-\varphi)c\gamma n_1 n_2$ is worth V per unit of well-being today, and provides the human population growth for tomorrow. The share to trade $sc\gamma n_1 n_2$ is exported for per-unit current benefit and, as we assume proceeds cannot be used to affect the resource base or its harvest, reduces the available resource for future growth in capital (elite) or commoner population while replacing it with non-resource-based consumption goods and services. The share to the elite (capital) $\varphi(1-s)c\gamma n_1 n_2$ creates current value as well as future value through investment in the resource base and human population. The sum total determines the remaining resource base available for growth (or replenishment) and future value.

A1.2.1 Value from laborer consumption

The marginal value of consumption of the resource for human subsistence by commoners/laborers is generally represented as a decreasing benefit (utility) function $\left(V\left(\psi, I_t\right)d\psi\right)$, which we simplify to an exogenous benefit value, V_t.

In addition to the current benefits of consumption, $V^*(1-s)(1-\varphi)c\gamma n_1 n_2$, the current harvest affects the future resource base and human commoner population. The resource base grows according to a standard logistic growth function net of harvest, described further below. The human laborer population tomorrow is a function of the ability to convert consumption to growth (via an intrinsic growth rate), the death rate of the population $\left(d_t\right)$, and the intraspecific rate of competition, i.e. the rate at which members of the laborer population compete for the same resources, $\chi\left(K_2, n_2, t\right) = \chi\left(K_2, t\right)$. $\chi = 0$ implies that there is no deadly competition for the resource base, so it must be that the population can simply expand with extensive growth (e.g. into new agricultural lands). For our purposes, the coefficient is fixed with respect to human commoner population[29] but is a function of the elite (capital), where the assumption that $\frac{\partial \chi}{\partial K_2} \leq 0$ implies capital accumulation (through an elite) can counteract crowding by resource-increasing investment and an expanding production possibilities frontier. The death rate could similarly be modeled as an explicit function of K_2 or export price, but here we simply allow it to vary exogenously over time.

A1.2.2 Value from Capital/Elite formation

The marginal benefit (utility) of capital accumulation is described generally here as a decreasing benefit function, $V_{K_2 t}(\zeta)d\zeta$, where total benefit from elite formation (capital) is the integral of this function over the level of capital. Below, we simplify to an exogenous value, P_{kt}, that can be shifted by changes in the benefits of wealth (perhaps prestige, power, or access to luxury goods). The share to capital, $\varphi(1-s)c\gamma n_1 n_2$, contributes to current well-being through a technical transformation (A^{ξ}) from the resource into capital allowing for $A^{\xi}\varphi(1-s)c\gamma n_1 n_2$ units of new capital formation. Further, capital contributes to future value through its use in increasing the resource carrying capacity (e.g. organizing construction of irrigation, fishponds) and reducing the

[29] $\chi'(n_2) \geq 0$ would imply the greater the population the greater the competition (i.e. when extensive growth used up, intensive growth pressures grow stronger).

intraspecific competition coefficient (e.g. opening new areas to settlement, resolving disputes). The opportunity costs of the elite (capital) are enforcement and the reduction in resource availability for trade or commoner consumption (and population growth). We hold technology constant here for simplicity, but one could expand the model further by allowing capital investment or exogenous shifts to change this function as well. We presume that investment and/or contact with others through trade could increase the ability to transform the resource into capital value and would thus increase the amount of capital available to the system.

A1.2.3 Value from trade

The marginal benefit (utility) of the resource as an export commodity is represented by a downward sloping demand function, $D_{st}^{-1}(\zeta, I_t)\, d\zeta$. We simplify this to an exogenous price, P_{st}, that can be shifted by changes in opportunities for trade. The current net benefits to trade are the net value, $(P_s - w_s) sc\gamma n_1 n_2$, which must be balanced against the lack of availability of that resource for either consumption or capital purposes so that human laborer population and capital (elite) growth will be lower with more trade, unless trade replaces the lost resource base with new opportunities. Control of the resource for trade and the distribution of returns from trade are then important factors in support for the institutional structure of the economy.

A1.3. Costs

These benefits are countered by the costs of the harvest and costs of harvest governance, which apply regardless of end use of the resource. Here we discuss the relationships between marginal costs and the working of the resource dependent system. Fixed costs, and their effect on the rate and magnitude of change of the marginal governance costs, are expected to be an important component of shifts amongst institutions, but we do not explore this more deeply in the formal model because the comparisons between institutions require comparing the results of the model derived under these different institutional structures, which is more simply accomplished in the discussion. Note that all of these costs may also change through exogenous shocks over time.

A1.3.1. Current Harvest Costs, Including Harvest Governance

The per-unit cost of harvest may be a function of the resource population and/or capital stock, but it is modeled here as exogenous $\left(w_c\left(n_{1t}, K_{2t}, t\right) = w_{ct}\right)$ for simplicity. This is because effects of changes in the per unit governance costs of the catch, $w_\gamma(\gamma_t, n_{1t}, n_{2t}, \mathrm{t}; \mathrm{I}_t)$, and enforcement

costs of the shares to capital accumulation, $w_\varphi(\varphi_t, n_2, \mathrm{t}; I_t)$, and trade, $w_s(\mathrm{s}_t, n_2, \mathrm{t}; I_t)$ (described below), are expected to behave very similarly. Furthermore the dynamics of endogenous harvest costs are well-explored in the fisheries literature.

Also because the differing enforcement costs have overlap in their expected behavior, and therefore to simplify exposition, we simplify unit governance costs of the stock so that $w_\gamma(\gamma_t, n_{1t}, n_{2t}, \mathrm{t}; \mathrm{I}_t) = w_\gamma(\gamma_t, n_{1t}, \mathrm{t})$. In other words increases in the human laborer population do not have an effect on the unit governance costs. The governance costs are modeled as a non-increasing function of the resource population, $\frac{\partial w_\gamma}{\partial n_1} \leq 0$. This follows from the fact that the more of the resource available in a location, the cheaper the unit costs of monitoring of the stock. Thus if we consider the location to be the area of suitable resource habitat conditions, then increasing the resource population serves to decrease the marginal governance costs.

The unit governance costs are assumed to be non-decreasing in governance, (recall $\gamma = 0$ is the tightest governance, restricting harvest to zero) so that we assume $\frac{\partial w_\gamma}{\partial \gamma} \leq 0$ as it becomes more costly to control the catch the more control is applied. It is much more costly, for example, to patrol a fishery 24 hours a day than just during daylight hours.

A1.3.2. Current Enforcement Costs of Non-consumption

We model the costs of enforcing non-laborer-consumption separately for capital and trade to allow flexibility in considering how shifts in external prices for the resource base may create different pressures and costs on enforcement and to better reflect on the role of management (through a non-productive governing elite). We assume that costs of portioning off the share to the elite, or capital accumulation, $w_\varphi(\varphi_t, n_2, \mathrm{t})$, are non-decreasing in φ, so that $\frac{\partial w_\varphi}{\partial \varphi} \geq 0$. This is because the more of the resource that is taken from laborer consumption, the greater the monitoring costs and related costs of ensuring that the capital is efficiently allocated. We also assume that $w_\varphi(\varphi_t, n_2, \mathrm{t})$ is non-decreasing in the number of people needing to cooperate, as one expects in commons problems, so that $\frac{\partial w_\varphi}{\partial n_2} \geq 0$. For simplicity, costs of enforcing a share to trade, $w_s(\mathrm{s}_t, n_2, \mathrm{t}; I_t) = w_s(n_2, \mathrm{t})$ are assumed to be linear in the share to trade. They could instead be

modeled as increasing for the same reasons as drive our assumption that $\frac{\partial w_\varphi}{\partial \varphi} \geq 0$. Costs of enforcing a share to trade are modeled as non-decreasing with population levels $\frac{\partial w_s}{\partial n_2} \geq 0$. We assume this because the opportunity cost of trade over consumption will increase and/or more individuals involved in trade result in more parties to monitor who might rather consume the resource.

A1.4 Resource Harvest

The resource (n_{1t}) is harvested by the population, n_{2t}, at a per capita rate of $c(K_{2t}, n_{1t}, \gamma(I_t))n_{1t}$, where $c(K_{2t}, n_{1t}, \gamma(I_t)) = c \in [0,1]$. This rate is a standard bio-economic catchability coefficient, or in other words the ability of the population to convert a unit of resource into sustenance (for example the catch rate for fish or the hunting success rate of a pig population), with two additional characteristics. These are that governance, in the form of harvest restrictions, may lower the coefficient, and that the coefficient may be a function of capital. With respect to the latter characteristic, we in general expect increases in capital investment (and/or harvest technology) to increase catchability of the resource population, n_{1t}. We in general assume that the more abundant the resource, the easier the harvest, as one would expect. We simplify the presentation by assuming that the capital accumulation/elite (K_{2t}) can grow the resource carrying capacity, K_{1t}, and therefore the resource stock, but does not directly affect catchability, so that $c(K_{2t}) = c_t$, while the resource carrying capacity is $K_{1t}(K_{2t})$. As stated, the harvest level, via the catchability coefficient, is also a function of governance of the stock, $\gamma(I_t)$, so that increased governance (increases in $1-\gamma_t$) can reduce the harvest.

We simplify the relationships between populations and governance by allowing only the resource population and the intensity of governance to affect per unit governance costs $(w_{\gamma t}(\gamma_t, n_{1t}, t))$ of the harvest. We make this assumption since human population will similarly but more directly affect the costs of enforcement ($w_\varphi(\varphi_t, n_{2t}, t)$ and $w_s(n_{2t}, \mathrm{t})$) of the shares not for consumption $\varphi_t(1-s_t)+s_t$ .by the local human population (n_2), since these do not operate through the catchability coefficient.

A1.5 Formal statement of the model

In its most general form, we write the value of social welfare over time as:

$$\int_{t=0}^{\infty} e^{-rt} \left(\begin{array}{l} \underbrace{\int_0^{K_{2t}} V_{K_{2t}}(\zeta, I_t)\, d\zeta}_{value\ of\ capital} + \underbrace{\int_0^{s_t c_t \gamma_t n_{1t} n_{2t}} D_{st}^{-1}(\omega, I_t)\, d\omega}_{value\ of\ export} + \underbrace{\int_0^{(1-s_t)(1-\varphi_t) c_t \gamma_t n_{1t} n_{2t}} V(\psi, I_t)\, d\psi}_{value\ of\ consumption} \\ -\left(\underbrace{w_c(n_{1t}, K_{2t}, t)}_{direct\ unit\ harvest\ cost} + \underbrace{w_\gamma(\gamma_t, n_{1t}, n_{2t}, \mathrm{t}; \mathrm{I}_t)(1-\gamma_t(I_t))}_{unit\ costs\ of\ governance} + \underbrace{w_\varphi(\varphi_t, n_2, \mathrm{t}; I_t)\varphi_t(I_t) + w_s(s_t, n_2, t; I_t)\mathrm{s}_t(I_t)}_{unit\ costs\ of\ non\text{-}laborer\ consumption} \right) \\ * \underbrace{c(K_{2t}, n_{1t}, \gamma(I_t), t)\, n_{1t} n_{2t}}_{harvest} \end{array} \right) dt$$

Using the simplifications discussed above, the maximization problem can be written as:

$$Max \int_{t=0}^{\infty} e^{-rt} \left(P_{kt} K_{2t} + \left(\begin{array}{l} P_{st} s_t + (1-s_t)(1-\varphi_t) V_t - w_{ct} \\ + w_\gamma(\gamma_t, n_{1t}, \mathrm{t})(1-\gamma_t) + w_\varphi(\varphi_t, n_2, \mathrm{t})\varphi_t + w_s(n_2, t)\mathrm{s}_t \end{array} \right) c_t \gamma_t n_{1t} n_{2t} \right) dt$$

subject to equations of motion for the resource and laborer populations and the evolution of the elite (capital stock) as shown in equations A1.1, A1.2, and A1.3 below.

The equation of motion for the resource is:

$$\dot{n}_1 = \underbrace{g_1 n_{1t} \left(1 - \frac{n_{1t}}{K_{1t}(K_{2t})} \right)^{\sigma}}_{natural\ growth} - \underbrace{(\gamma_t c_t n_{1t}) n_{2t}}_{harvest} \qquad \text{A1.1}$$

The resource population n_1 follows a logistic growth function with a human population-dependent harvest. Here g_1 is the intrinsic growth rate of the resource, σ is the density dependence of the resource stock (assumed 1 for simplicity in our case), and K_1, the carrying capacity at t of the resource, can be increased through capital accumulation under the assumption that $K_1' \geq 0$. For expositional purposes we choose a simple relationship where $K_{1t}(K_{2t}) = \underline{K}_1 + k_t K_{2t}$. Here $\underline{K}_1$ is the natural carrying capacity of the resource and $k_t \geq 0$ is a time-varying parameter for transforming the current capital stock into carrying capacity increase.

The equation of motion for the human laborer (commoner) population is:

$$\dot{n}_2 = n_{2t}\left(\underbrace{g_2\left(1-\varphi_t\right)\left(\gamma_t c_t n_{1t}\left(1-s_t\right)\right)}_{resource\ dependent\ natural\ growth} - \underbrace{d_t}_{death\ rate} - \underbrace{\chi\left(K_{2t},\mathrm{t}\right)n_{2t}}_{intraspecific\ competition}\right) \quad \text{A1.2}$$

Where g_2 is the intrinsic growth rate (conversion efficiency) of the human laborer population, d = death rate of humans, which could be modeled as an explicit function of K_2 or export price, but here we simply allow to vary exogenously over time, and $\chi\left(K_2,n_2,t\right)=\chi\left(K_2,t\right)$ is the intraspecific competition coefficient at t. $\chi=0$ implies the commoner population can just expand with extensive growth (e.g. into new agricultural lands). For our purposes, the coefficient is fixed with respect to human commoner population[30] but a function of the elite (capital), where the assumption that $\frac{\partial\chi}{\partial K_2}\leq 0$ implies the elite (capital accumulation) can counteract crowding by resource-increasing investment and an expanding production possibilities frontier.

Finally, the equation of motion for capital accumulation or support of an elite is:

$$\dot{K}_2 = K_{2t} + \underbrace{A_t^{\xi}\varphi_t\left(\gamma_t c_t n_{1t}\left(1-s_t\right)\right)n_{2t}}_{investment} - \delta K_{2t} \quad \text{A1.3}$$

Where A is the technology coefficient at t, which we assume to be fixed, along with the growth parameter ξ, for simplicity, and δ is the (fixed) depreciation rate of capital (e.g. death rate for the elite).

We generate a current value Hamiltonian using $\lambda_t, v_t,$ *and* ϑ_t as the co-state variables for the equations of motion for the resource, human population, and capital respectively and take the subsequent relevant first order conditions to generate the necessary conditions (time subscripts suppressed) for dynamic optimization to be:

$$\dot{n}_1 = \frac{\partial H}{\partial\lambda} = g_1 n_1\left(1-\frac{n_1}{\underline{K}_1 + kK_2}\right) - \gamma c n_1 n_{2t} \quad \text{A1.4}$$

---

30 $\chi'(n_2)\geq 0$ would imply the greater the population the greater the competition (i.e. when extensive growth used up, intensive growth pressures grow stronger).

$$\dot{n}_2 = \frac{\partial H}{\partial \nu} = n_{2t}\left(g_2\left(1-\varphi\right)\left(\gamma c n_1\left(1-s\right)\right) - d - \chi\left(K_2\right)n_2\right) \quad \text{A1.5}$$

$$\dot{K}_2 = \frac{\partial H}{\partial \vartheta} = K_2 + A^{\xi}\varphi\left(\gamma c n_1\left(1-s\right)\right)n_2 - \delta K_2 \quad \text{A1.6}$$

$$\frac{\partial H}{\partial \gamma} = c n_1 n_2 \begin{pmatrix} \left(P_s - w_s\right)s + \left(1-s\right)\left(1-\varphi\right)V \; - w_{ct} - \varphi w_{\varphi} - \left(1-\gamma\right)w_h \\ +A_t^{\xi}\varphi\left(1-s\right)\vartheta - \lambda + g_2\left(1-\varphi\right)\left(1-s\right)\nu - \gamma\left(-w_{\gamma} + \left(1-\gamma\right)\dfrac{\partial w_{\gamma}}{\partial \gamma}\right) \end{pmatrix} = 0$$

A1.7

$$\frac{\partial H}{\partial \varphi} = \left(A^{\xi}\vartheta - g_2\nu\right)\left(ch = \gamma n_1 n_2\left(1-s\right)\right) - c\gamma n_1 n_2\left(-\left(1-s\right)V + w_{\varphi} + \frac{\partial w_{\varphi}}{\partial \varphi}\varphi\right) = 0 \quad \text{A1.8}$$

$$\frac{\partial H}{\partial s} = \left(P_s - \left(1-\varphi\right)V - w_s\right) - \vartheta\left(A^{\xi}\varphi\right) - \nu\left(g_2\left(1-\varphi\right)\right) = 0 \quad \text{A1.9}$$

$$\frac{\partial H}{\partial n_{1t}} = r\lambda_t - \dot{\lambda} =$$

$$c\gamma n_2\left(P_s s + \left(1-s\right)\left(1-\varphi\right)V - w_c - \varphi w_{\varphi} - (1-\gamma)w_h - s w_s + A^{\xi}\varphi\left(1-s\right)\vartheta + g_2\left(1-\varphi\right)\left(1-s\right)\nu - \left(1-\gamma\right)n_1\frac{\partial w_{\gamma}}{\partial n_1}\right)$$

$$-\left(c\gamma n_2 + \left(\frac{2n_1}{\underline{K}_1 + \kappa K_2} - 1\right)g_1\right)\lambda$$

A1.10

$$\frac{\partial H}{\partial n_{2t}} = r\nu - \dot{\nu} = c\gamma n_1\left(P_s s + \left(1-s\right)\left(1-\varphi\right)V - w_c - \varphi w_{\varphi} - \left(1-\gamma\right)w_h - s w_s + \varphi A^{\xi}\left(1-s\right)\vartheta - \lambda - n_2\left(s\frac{\partial w_s}{\partial n_2} + \varphi\frac{\partial w_{\varphi}}{\partial n_2}\right)\right)$$

$$-\nu\left(-d + g_2\left(1-\varphi\right)\left(c\gamma n_1\left(1-s\right)\right) - 2n_2\chi\right)$$

A1.11

$$\frac{\partial H}{\partial K_2} = r\vartheta - \dot{\vartheta} = p_{K_2} + \left(1-\delta\right)\vartheta + \frac{g_1\kappa n_1^2}{\left(\underline{K}_1 + \kappa K_2\right)^2}\lambda - n_2^2\frac{\partial \chi}{\partial n_2}\nu \quad \text{A1.12}$$

A1.6 Model solutions

Through rearrangement and substitution, we first find expressions for the co-state variables are:

$$\lambda = \left(P_s - w_s\right) - w_c - \varphi w_{\varphi} - (1-2\gamma)w_{\gamma} - \gamma(1-\gamma)\frac{\partial w_{\gamma}}{\partial \gamma} \quad \text{A1.13}$$

$$\nu = \frac{(P_s - w_s) - (1-2\varphi)V}{g_2} - \frac{\varphi}{g_2(1-s)}\left(w_\varphi + \varphi\frac{\partial w_\varphi}{\partial\varphi}\right) \qquad \text{A1.14}$$

And

$$\vartheta = \frac{(P_s - w_s) - (1-2\varphi)V}{A^\xi} + \frac{(1-\varphi)}{(1-s)A^\xi}\left(w_\varphi + \varphi\frac{\partial w_\varphi}{\partial\varphi}\right) \qquad \text{A1.15}$$

From Eqns A1.14 and A1.15 we see that the shadow values are functions of the ability to transform the resource harvest into population and capital growth, respectively. Using these equations and their time derivatives with equations A1.10-A1.12, we can solve for a set of intertwined optimal extraction paths for the resources. Equations A1.16- A1.18 weigh the discount rate against growth in the three different arenas: the resource base, the human laborer population, and capital accumulation (elite formation). One can readily see that the basic conditions for optimal growth are augmented by marginal stock effect terms that are dependent on the expected net benefits of the varying governance options, the harvest costs, and the value of external trade (P_s):

$$r = g_1\left(1 - \frac{2n_1}{\underline{K}_1 + kK_2}\right) + \frac{c\gamma n_2\left(-\gamma w_\gamma + (1-\gamma)\left(w_\gamma + n_1\frac{\partial w_\gamma}{\partial n_1}\right)\right) + \dot{P}_s + 2V\dot{\varphi} - (1-2\varphi)\dot{V} - \dot{w}_s - \dot{w}_c - \dot{\varphi}\left(w_\varphi + \frac{\partial w_\varphi}{\partial\varphi}\right) - \varphi\dot{w}_\varphi + \dot{\gamma}\left(2w_\gamma - 2\gamma\frac{\partial w_\gamma}{\partial\gamma} + \frac{\partial^2 w_\gamma}{\partial\gamma^2}\right) + \dot{n}_1\left(\frac{\partial w_\gamma}{\partial n_1} + \frac{\partial^2 w_\gamma}{\partial\gamma\partial n_1}\right) - \dot{n}_2\left(\frac{\partial w_s}{\partial n_2} + \varphi\frac{\partial w_\varphi}{\partial n_2}\right)}{P_s - w_s + (1-s)(1-\varphi)V - w_c - \varphi w_\varphi + (2\gamma-1)w_\gamma - (1-\gamma)\frac{\partial w_\gamma}{\partial\gamma}}$$

A1.16

In Eqn A1.16, we see that growth should follow a path set by the natural growth capabilities of the resource and a marginal stock effect. In the numerator of the marginal stock effect is $c\gamma n_2$, the per-resource unit contribution to human population derived from the resource, multiplied by a term reflecting the effect of governance of the harvest and its costs. The denominator adjusts the marginal stock effect for the value of this contribution, including costs of enforcement and governance.

In Eqn A1.17, we see that the second path similarly reflects that the growth of the human population, net of a marginal stock effect, should match the discount rate. The marginal stock effect in this case describes the marginal effect on the population from consumption of the changing resource stock: it equals $c\gamma n_1 g_2$, the per-person growth from consumption of the resource stock, as impacted by the governance costs of the share to the elite/capital, the governance costs of the harvest, and the change in costs of enforcement due to changes in the laborer population, scaled by the resource value in the denominator.

$$r = d + 2n_2\chi + \frac{g_2 c\gamma n_1\left(\left(2\varphi(1-\varphi)\left(w_\varphi+\varphi\frac{\partial w_\varphi}{\partial\varphi}\right)-w_\gamma\gamma+\gamma(1-\gamma)\frac{\partial w_\gamma}{\partial\gamma}-n_2\left(s\frac{\partial w_s}{\partial n_2}+\varphi\frac{\partial w_\varphi}{\partial n_2}\right)\right)\right)+\dot{P}_s-\dot{w}_s+\dot{n}_2\frac{\partial w_s}{\partial n_2}-\frac{\left(w_\varphi+\varphi\frac{\partial w_\varphi}{\partial\varphi}\right)\left(\dot{\varphi}(1-s)+\varphi\dot{s}\right)}{(1-s)^2} + \left(\frac{-\varphi}{1-s}\right)\left(\dot{w}_\varphi\left(1+\varphi\frac{\partial w_\varphi}{\partial\varphi}\right)+\dot{n}_2\left(\frac{\partial w_\varphi}{\partial n_2}+\varphi\frac{\partial^2 w_\varphi}{\partial\varphi\partial n_2}\right)+\dot{\varphi}\left(2\frac{\partial w_\varphi}{\partial\varphi}+\varphi\frac{\partial^2 w_\varphi}{\partial\varphi^2}\right)\right)}{P_s-w_s+(1-s)(1-\varphi)V-\left(\frac{\varphi}{1-s}\right)\left(w_\varphi+\varphi\frac{\partial w_\varphi}{\partial\varphi}\right)}$$

A1.17

Finally, in equation A1.18 we see that growth should follow capital accumulation with deterioration, net of a marginal stock effect. The marginal stock effect has several components that relate the changes in the human and resource populations on which the capital depends to the changes in value to the society through the technological transformation of the capital to growth. We examine these relationships at greater length in the text, with some more mathematical details of the exposition here.

$$r=\left(1-\delta\right)+\frac{A^{\xi}\left(P_{K_2}-\left(P_s-w_s\right)\frac{n_2^2\frac{\partial\chi}{\partial n_2}}{g_2}+\left(\dot{P}_s-\dot{w}_s\right)+\frac{g_1 k n_1^2}{\left(\underline{K}_1+kK_2\right)^2}\left(\left(P_s-w_s\right)+(1-s)(1-\varphi)V-w_c-\varphi w_\varphi+\left(2\gamma-1\right)w_\gamma-\gamma\left(1-\gamma\right)\frac{\partial w_\gamma}{\partial\gamma}\right)\right)+\left(w_\varphi+\varphi\frac{\partial w_\varphi}{\partial\varphi}\right)\left(A^{\xi}\frac{n_2^2\frac{\partial\chi}{\partial n_2}\varphi}{g_2\left(1-s\right)}+\frac{(1-\varphi)\dot{s}-\dot{\varphi}(1-s)}{\left(1-s\right)^2}\right)+\frac{\dot{n}_2}{\left(1-s\right)}\left(\left(-(1-s)\frac{\partial w_s}{\partial n_2}+\varphi\frac{\partial^2 w_\varphi}{\partial\varphi\partial n_2}\right)+A^{\xi}\varphi\left(\dot{w}_\varphi+\dot{\varphi}\left(2\frac{\partial w_\varphi}{\partial\varphi}+\varphi\frac{\partial^2 w_\varphi}{\partial\varphi^2}\right)\right)\right)}{P_s-w_s+\left(1-s\right)\left(1-\varphi\right)V+\frac{\left(1-\varphi\right)}{\left(1-s\right)}\left(w_\varphi+\varphi\frac{\partial w_\varphi}{\partial\varphi}\right)}$$

A1.18

A1.6 Phases of Growth Illustrated by the Model

A1.6.1 Extensive Growth

We highlight governance's role in extensive growth from the model by simplifying the system so that there is no value from non-labor-consumptive uses, so that $P_{K_2}=k=P_s=0$. No shares will then be devoted to non-labor-consumptive uses, since they will be costly with no return, thus $\varphi=s=0$. First, from (Eqn. A1.17), we obtain a condition for harvest governance where:

$$V\frac{\left(d+2\chi n_2-r\right)}{g_2 c\gamma n_1}=\gamma\left(w_\gamma-\left(1-\gamma\right)\frac{\partial w_\gamma}{\partial\gamma}\right) \qquad \text{(A1.19)}$$

The right hand side is the present value of the marginal benefit of governance, which is equal to the marginal cost of governance. If governance is not used, so that there is open access ($\gamma=1$) as a corner solution, it must be that $V\frac{\left(d+2\chi n_2-r\right)}{g_2 c n_1}\le w_\gamma$ for open access to be efficient. In other words, if the per-person return on the value of the resource (marginal benefit of governance) is less than or equal to the marginal governance costs, open access is efficient. There is no a priori reason to assume this condition could not hold.

Rearranging A1.19, we see how the optimal stock of the resource and the population are simultaneously determined. The optimal stock of the resource is

$n_1^* = \dfrac{V\left(d + 2\chi n_2^* - r\right)}{g_2 c\gamma^2\left(w_\gamma - (1-\gamma)\dfrac{\partial w_\gamma}{\partial \gamma}\right)}$ as long as $d + 2\chi n_2^* > r$. This condition suggests that if the future is worth little ($r > d + 2\chi n_2$), particularly in comparison to losses in the population from death and competition, a stable steady state population greater than zero will not occur.

Since $\dfrac{\partial w_\gamma}{\partial \gamma} \leq 0$, an increase in governance $(1-\gamma)$ will increase the denominator and increase the optimal resource population. Note also that the greater the ability to convert the resource base to human population or to harvest the resource (the higher g_2 or *c*) then the lower the optimal resource base. A higher death rate or greater intraspecific competition increases the resource population (with fewer mouths to feed) and higher value placed on the future lowers the discount rate *r* and increases the resource population, as expected. We argue that this specification captures extensive growth from an initially small human population exploiting a newly available resource, with governance of the harvest providing the ability to increase the resource population.

A1.6.2 Intensive Growth and Capitalization

As the human population expands to reach the limitations of a resource and human population where there is no room for further growth to evolve, we consider how the formation of an elite, or capital investment, may change the equation. We continue to limit opportunities for trade so that $P_s = s = 0$. Now, however, capital accumulation has a positive value as investment into expanding the resource's carrying capacity so that $k > 0$ and $P_{K_2} > 0$. We investigate how the incentive to set aside a portion of today's laborer consumption affects the resource base and human laborer population in relation to the case of extensive growth. The modification to a steady state condition on populations, if it exists, from equation A1.17 is:

$$n_1^* = \frac{\left(d + 2\chi n_2^* - r\right)\left((1-\varphi)V - \varphi\left(w_\varphi + \varphi\dfrac{\partial w_\varphi}{\partial \varphi}\right)\right)}{g_2 c\gamma\left(\gamma\left(w_\gamma - (1-\gamma)\dfrac{\partial w_\gamma}{\partial \gamma}\right) + \varphi\left(2(1-\varphi)\left(w_\varphi + \varphi\dfrac{\partial w_\varphi}{\partial \varphi}\right) - n_2^*\dfrac{\partial w_\varphi}{\partial n_2}\right)\right)}. \qquad \text{(A1.20)}$$

We see then that the costs of enforcing that a portion of the resource goes to a governing elite (capital) have multiple channels through which they affect populations. On the one hand, increasing the share to the elite (capital) decreases the optimal laborer population level, as one would expect, through the numerator. This effect comes both from the higher share to the elite and from higher governance costs expected with the share, as $\frac{\partial w_\varphi}{\partial \varphi} \geq 0$. Since $\varphi \in [0,1]$, the net effect of the first denominator term containing it will be to decrease the optimal laborer population, in agreement with the numerator effects. If there is no impact of the human laborer population on the cost of enforcement of the elite, then the overall impact of using a share of the harvest is to decrease the optimal population. However as $\frac{\partial w_\varphi}{\partial n_2} > 0$ then the effect on the cost of enforcement of the elite depends on first on whether n_2 is increasing or not, since if it is decreasing then the final term is growing smaller and the denominator increases, also reducing the resource base. If, however, the effect on the population from lowering the resource base is to increase it, then this mitigates the pressures from the elite on the resource base. Since a lower resource base indicates a higher catch, the question of whether the human laborer population is increasing or decreasing depends on the magnitude of the share to the elite (capital accumulation) relative to the case of extensive growth without capital accumulation.

We have additional information about the system from the other optimal extraction paths. We find first that

$$\frac{A^\xi P_{K_2}}{r+\delta-1} = (1-\varphi)\left(V + \left(w_\varphi + \varphi \frac{\partial w_\varphi}{\varphi}\right)\right), \qquad \text{(A1.21)}$$

i.e. that the present value of a unit of the resource converted to capital (the elite) is equal to it marginal opportunity costs: the current value of a unit portion of laborer consumption plus a combined term reflecting the direct and indirect costs of enforcement of the share to the elite. Note that the cost of harvesting the resource itself, w_c, is not part of the tradeoff here as it must be paid regardless of the use of the resource. The elite, in essence, must pay for itself with returns to governance.

Further, we can express the share to the elite (capital) if there is a steady state as:

$$\varphi = \frac{\left(V - w_c + (2\gamma - 1)w_\gamma - (1-\gamma)\frac{\partial w_\gamma}{\partial \gamma}\right) - \left(\frac{c\gamma n_2\left(\left(1-2\gamma\right)w_\gamma + (1-\gamma)n_1\frac{\partial w_\gamma}{\partial n_1}\right)}{r - g_1\left(1 - \frac{2n_1}{\underline{K}_1 + kK_2}\right)}\right)}{w_\varphi + V}. \tag{A1.22}$$

This we interpret as a cost weighted unit share of the return from the harvest to population (the first combined term in the numerator) net the loss in growth that will occur in population growth from moving the marginal unit to capital (the second combined term in the numerator). The share is unsurprisingly decreasing in the cost of enforcing the share. The easier the conversion of the resource stock to laborer population (from the 2[nd] combined numerator term), the greater is the share of the resource that can go to capital accumulation (the elite). The larger the ability to transform capital into increased carrying capacity (the larger the value of *k*), the lower the share needed to go to capital, so φ is decreasing in *k* as well as the original carrying capacity, $\underline{K}_1$, and K_2.

The effect of an increase in human laborer population is to increase the share to capital if $n_1 > \frac{\left(2\gamma - 1\right)w_\gamma}{(1-\gamma)\frac{\partial w_\gamma}{\partial n_1}}$. This will certainly be the case if $\gamma > ½$ since this renders the RHS of the formula negative. Thus more lax governance of the harvest itself, supporting a larger current laborer population, can be counterbalanced by a larger share to the elite (capital), which in turn expands the resource carrying capacity.

A1.6.3 External Trade

The full specification of the model then includes opportunities for external trade. For ease of exposition, we first investigate the role of trade in the model by hypothesizing that there is no value for capital accumulation so that $\varphi = P_{K_2} = K_2 = 0$. We solve for the share of the resource that should go to trade at a steady state, if it exists:

$$s = \frac{\left(\frac{\left(d + 2n_2\chi - r\right)}{g_2 c\gamma n_1}\left(P_s - w_s + V\right) + \left(-w_\gamma\gamma + \gamma\left(1-\gamma\right)\frac{\partial w_\gamma}{\partial \gamma}\right)\right)}{n_2\frac{\partial w_s}{\partial n_2} + V\frac{\left(d + 2n_2\chi - r\right)}{g_2 c\gamma n_1}}. \tag{A1.23}$$

We see that the share to trade, in addition to depending on the ability of the system to convert the resource into value either for trade or consumption, should be lower the higher the laborer population and the greater the effect of the laborer population on the enforcement costs of

guaranteeing a share for trade, as $\frac{\partial w_s}{\partial n_2} \geq 0$ by assumption. Among other things, if an institutional shift means that $\frac{\partial w_s(I_1)}{\partial n_2(I_1)} > \frac{\partial w_s(I_2)}{\partial n_2(I_2)}$, then the share to trade can grow with a shift from I_1 to I_2, ceteris paribus.

We find a second simultaneous expression for the share that is derived from the conditions pertaining to the resource base:

$$s = 1 + \frac{P_s - w_s - w_c + (2\gamma - 1) w_\gamma \gamma - (1-\gamma)\frac{\partial w_\gamma}{\partial \gamma}}{V} - \frac{c\gamma n_2 \left(-w_\gamma \gamma + (1-\gamma)\left(w_\gamma + n_1 \frac{\partial w_\gamma}{\partial n_1} \right) \right)}{\left(r - g_1 \left(1 - \frac{2n_1}{\underline{K}} \right) \right) V}. \qquad \text{(A1.24)}$$

The share s is increasing in the relative value of trade (the second combined term) and decreasing in the cost to the population. Note that the entire harvest will go to trade if there is no cost to the human population as long as the value from trade is positive.

For both the relationships in equations (4) and (5) to hold, it must be that

$$V = \frac{c\gamma n_2 \left(w_\gamma \gamma - (1-\gamma)\left(w_\gamma + n_1 \frac{\partial w_\gamma}{\partial n_1} \right) \right) + n_2 \frac{\partial w_s}{\partial n_2}\left(-w_c + (2\gamma-1) w_\gamma \gamma - (1-\gamma)\frac{\partial w_\gamma}{\partial \gamma} \right)}{\left(\left(g_1 \left(1 - \frac{2n_1}{\underline{K}} \right) - r \right) \left(\left(w_\gamma \gamma - \gamma(1-\gamma)\frac{\partial w_\gamma}{\partial \gamma} \right) - n_2 \frac{\partial w_s}{\partial n_2} - \left(\frac{(d + 2n_2\chi - r)}{g_2 c\gamma n_1} \right) \left(-w_c + (2\gamma-1) w_\gamma \gamma - (1-\gamma)\frac{\partial w_\gamma}{\partial \gamma} \right) \right) \right)}$$

(A1.25)

This is the present value marginal cost per unit of resource population from the governed catch, with trade. The numerator consists of the per-resource-unit costs of the governed harvest, including enforcement of the share to trade and the fact that governance costs vary with resource population and enforcement costs vary with human population. The denominator captures the opportunity cost of future growth, both from the resource population and the human population dynamics. This is independent of the value of the traded resource.

.

Appendix 2: Centralized Authority and Decision-Making: The *ahupua´a* system

The top-down management of the *ahupua´a* can be classified as common-property management, albeit more sophisticated than commonly described.[31] The *ahupua´a* provided everything "from *uka*, mountain, whence came wood, *kappa* for clothing, *olona*, for fish-line, ti-leaf for wrapping paper, *ie* for rattan lashing, wild birds for food, to the *kai*, sea, whence came *i´a*, fish, and all connected therewith" (Lyons, 1875). Both internal economies, e.g. in fishpond construction, and external economies were exploited. The strong hierarchical authority also allowed enforcement of conservation measures that reduced the depletion of natural resources.

Under the *ahupua`a* system, governance took the specific form of the *kapu* system. A kapu, or taboo, functioned in part by enlisting the gods' support in watching over resource exploitation. The fear of a god witnessing the breaking of a *kapu* and inflicting punishment certainly reduced enforcement costs, but did not eliminate them.[32]

The chiefs limited access during certain seasons by placing a *kapu* (taboo) on fishing.[33,34] The *kapu* were clearly conservation oriented; one of the most important *kapu* created alternating closed seasons for two species of primary import.[35] Other *kapu* closed fisheries during spawning seasons in particular.

The community worked under a gift-exchange system known as *ko kula ʻuka, ko kula kai*, where those upland traded with those on the sea. This allowed considerable expertise and specialization to develop as evidenced by the highly developed knowledge and skill amongst both fishermen and planters, and kept most economic transactions within the *ahupua´a*. The *ali´i* placed taxes on the *maka´ainana* (commoners) by requiring them to deliver commodities such as taro and to contribute labor, e.g. to the building of fishponds. Enforcement of the hierarchy rested in part on

---

[31] See e.g. the cases decribed in Ostrom, 1990.

[32] In 1824, C.S. Stewart noted in his published journal that he had seen a brackish fishpond "literally alive with the finest of mullet; the surface of the water is almost in a constant ripple from their motions; and hundreds can be taken at any time by a single cast of a small net." He attributes this to the success of the *kapu* and the fact that no one of rank had lived there lately (Dieudonne, 2002, p. 105). Alternatively, a 19th century Hawaiian historian wrote that pond caretakers could eat some fish species openly, "but others they would eat secretly" (Summers, 1964).

[33] Fishponds may have been a response to this resource pressure not only as a source of increased production, but also as a social mechanism by which the *ali´i* could continue to consume fish during the *kapu* periods without "offending the gods." Indeed, two main benefits arose from the ponds: (1) fish could be held and cultivated for easy access by the chiefs when desired, and (2) fish would be available to the chiefs during times of *kapu*, because the enclosure removed the area from the sea, which had the *kapu*, and placed it on land, from which the chiefs could still eat.

[34] These *kapu* are generally associated with particular gods and variants of the system are known throughout Polynesia.

[35] *ʻopelu* (Mackerel scad) and *aku* (skipjack tuna).

brutality and fear of the wrath not only of the chiefs but also of the gods. Both cultural conditions enhance the benefits of common property rights.[36]

Top-down management also allows the exploitation of benefits across ecosystem boundaries, not just within them. Some of these benefits fit the standard theory, such as increased risk reduction. However the *ahupua´a* system also provided the external economies of specialization and trade, e.g. between taro cultivators living on the plains and fishermen living on the coast. (Only external economies within the scope of the *ahupua'a* government could be readily exploited, however.)

The hierarchical system allowed exploitation of the external economies from specialization, given the existing avenues for trade, as well as internal economies in the production of particular goods. Furthermore, the centralized authority at the *ahupua´a* level satisfied the four requirements for viable common property rights discussed in the introduction. "Unambiguous property lines" prevailed in Hawaii as *ahupua´a* generally (though not always) followed watershed lines. "Economies of scale and ecosystem enhancement improved directly the lives of the people," as shown by investment in irrigation and fishpond infrastructure, increasing taro and fishery production capability, and the simultaneous existence of leisure-time; community property "alleviated risks of enemy incursion and reduced idiosyncratic risks" as a portion of the production of the *ahupua`a* was collected returned to the individuals through festivals, and planters and fishermen "retained portions of their effort, reaping individual benefits from their productivity."

The case of the *ahupua´a* system affords a further generalization to the condition that benefits should be rather equally divided across group members, i.e. proportional taxation can also be efficient and readily administered where wealth is unequally distributed, provided that separate rules are specified for each stratum and the members of each stratum have roughly equal entitlements.

First, the top-down management of the meant that work and reward were not distributed equally across society, only within each stratum. This facilitates a more general statement about the condition for successful common property management, namely that the allocation of costs conforms to the principle of benefit taxation, albeit within the prevailing system of vertical equity.

Appendix 3: Summary of the Hawaiian Record in Economic Context

[36] See also Deininger 2003 (p. 31) for a discussion of the role of culture in the enforcement of common property in another context.

Table A1 summarizes the variables used in the formal model.

Table A1 Variable list	
Variable	Meaning (All values at time t, time subscript suppressed)
γ	Harvest resource governance parameter (control variable)
s	Share of the resource to trade (control variable)
$\varphi(1-s)$	Share of the resource to capital (control variable)
$(1-s)(1-\varphi)$	Share of the resource to consumption
n_1	Resource population (state variable)
n_2	Human population (state variable)
K_{2t}	Capital (state variable)
d_t	Human population death rate
χ	Intraspecific rate of competition
A	Technology parameter
ξ	Technology growth parameter
I_t	Institutional structure
g_1	Intrinsic growth rate of the resource
g_2	Intrinsic growth rate of the human population
σ	Density dependence of the resource stock (assumed=1)
K_1	Resource carrying capacity, with improvements
$\underline{K}_1$	Natural (base) carrying capacity of the resource
k	Transformation parameter from capital to carrying capacity
P_{kt}	Unit value of capital
P_{st}	Unit value of traded resource
V_t	Unit value of consumption
w_{ct}	Marginal cost of resource harvest
w_γ	Marginal cost of resource governance
w_φ	Marginal cost of governing share to capital
w_s	Marginal cost of governing share to trade
c	Catchability coefficient
r	Social discount rate
δ	depreciation rate of capital
λ	Co-state variable for equation of motion for the resource
ν	Co-state variable for equation of motion for the human population
ϑ	Co-state variable for equation of motion for capital

Table A2: Evolution of Specialization and Production through Unification

Time Period	
	P: Population
	SMS: Social/Management Structure
	PS: Production and Specialization; Technological Change
	IP: Intensification of Production
	T: Trade
Extensive Growth (400-1100 AD)	P: Grows from less than 100 to around 20,000
	SMS: Ohana network; ancestral; little social stratification
	PS: Wide variety of fishing implements and adzes; little specialization, evolves to incipient form of Hawaiian 2-piece fishhook,
	IP: Introduction of new plants, pigs, dogs, rats; transformation of landscape to support Polynesian culture
	T: Little; trade within ohana network develops
Extensive and Intensive Growth (1100-1650 AD)	P: Grows from c. 20,000 to several hundred thousand (estimates vary widely, from 110,000 to 1 m).
	SMS: Ohana network; stratification increasingly evident (status goods growing)
	PS: Coastal intensification; new 1-piece fishhook introduced and becomes dominant; Inland extensive growth: Adzes fully standardized
	IP: Extensive growth dominant; Beginnings of irrigation and development of fishponds; increasing productivity yields in wet windward valley
	T: Specialized producers of adzes (Mauna Kea) , trade apparent Fishing gear increasingly standardized
Intensive growth and Capitalization (1650-1778)	P: Population growth slows, pop'n may even decline with increased warfare, labor taxes through hierarchy
	SMS: Transition to territorial hierarchy (*ahupua'a* system) complete (*konohiki* class evident, alii genealogy distinct from commoners now tied to land not family)
	PS: Craft specialists develop in producing status goods (feathers, carvings) for increasingly stratified *ali'I* class
	IP: Intensive dryland farming techniques developed; irrigation and fishpond development continues in established areas
	T: Increasing in the limited status goods (e.g. feathers from upland)

Trade: Economic growth under hierarchy (1778-1800)	P: Rapid decline in native population begins
	SMS: *Ali'i* and *kahuna* (priest) classes increasingly stratified, increasing intensity of *kapu*; *konohiki* managers and specialized commoners for land and sea; *ali'i* increasingly favor rent-seeking Increasing unification of authority
	PS: Introduction of western goods and technology increases efforts at crafts, shipbuilding, sandalwood harvest under *konohiki* management

	IP: Harvesting of resources for trade intensifies
	T: Western contact; trade for status goods and weapons

Table A3: Evolution of Production and Specialization, Post-Unification

Economic growth (trade) under Monarchy (1800-1840s)	P: c. 200,000, decreasing with disease, labor taxes, commoners leaving to work on ships
	SMS: Monarchy under Kamehameha I and family. Highly stratified, pressures on *kapu* increase with evidence from Western contact, rent-seeking; acquisition of status goods continues.
	PS: Development of use of Western goods (e.g. metal) and technology, mainly for extraction of rents
	IP: Increasing fishpond investment; intensive harvesting of sandalwood resource
	T: Continued accumulation of western goods among *ali'i*
Economic growth under Constitutional Monarchy (1840s-1900)	P: c. 90,000 (Population dwindles with western diseases, labor taxes)
	SMS: Constitutional monarchy; Private property initiated under Great Mahele; government and *konohiki* control marine property; public goods provision by state (e.g. education)
	PS: *Konohiki* managers become *konohiki* owners; Western interests accumulate land, introduction and growth of plantation sugar industry
	IP: Sandalwood depleted; fisheries suffering; fishpond development ends (1839 last pond)
	T: Heavy influence of small number of Westerners; sugar industry develops
Economic Growth as US Territory (c. 1900)	P: 154,000 (Foreign migration)
	SMS: Private ownership of land; increasing regulation of marine commons with size, gear restrictions; nearshore resources controlled by *konohiki*
	PS: Coastal fishing dwindles and offshore fishing increases in importance
	IP: Privately funded irrigation projects for sugar *Konohiki* owners balance enforcement benefits and costs as registration required to continue marine rights
	T: Islands become more dependent on imported goods as relative prices favor imported food, etc.
Economic Growth under Statehood (c. 2000)	P: 1.2 million
	SMS: Federal, State, and Local Controls Land: Privately owned with restrictions on use Fisheries: Regulated open access with subsidized fish populations
	PS: Diversified agriculture (plantation sugar industry unsustainable with removal of subsidies) Tourism (7 million visitors per year)
	IP: Public and private investment in cage farming technology Leasing of marine rights for cage farming -- intensification of fish production
	T: Islands dependent on trade

References

Deininger, K., 2003. Land Policies for Growth and Poverty Reduction Oxford: Oxford University Press.

Dieudonne, F., (Ed.) 2002. The Pacific Islands and the Sea: 350 Years of reporting on Royal Fishponds, Coral Reefs and Ancient Walled Fish Weirs in Oceania. Contributing Eds. J.M., Farber, S., Halapua. Neptune House Publications in association with Pacific Islands Development Program, East-West Center. Encinitas, California.

Lyons, C.J., 1875. Land matters in Hawaii. The Islander 118, 103-104.

Ostrom, E., 1990. Governing the Commons: Evolution of Institutions for Collective Action. Cambridge, Cambridge University Press. http://dx.doi.org/10.1017/CBO9780511807763

Summers, C., 1964. Hawaiian Archaeology: Hawaiian Fishponds. Honolulu, HI, Bishop Museum Press.